1

$A = 10^7$

$$F(s) = \frac{A}{(s+10)(s+10000)} = \frac{10^7}{(j\omega+10)(j\omega+10000)} = \frac{10^7}{10\left(\frac{j\omega}{10}+1\right)10^4\left(\frac{j\omega}{10^4}+1\right)} =$$

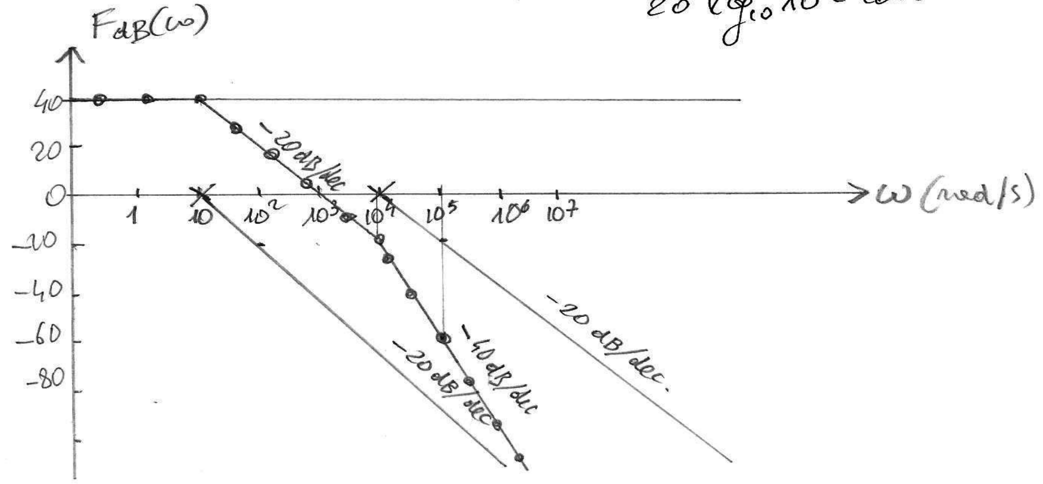
$$= \frac{10^2}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^4}\right)}$$

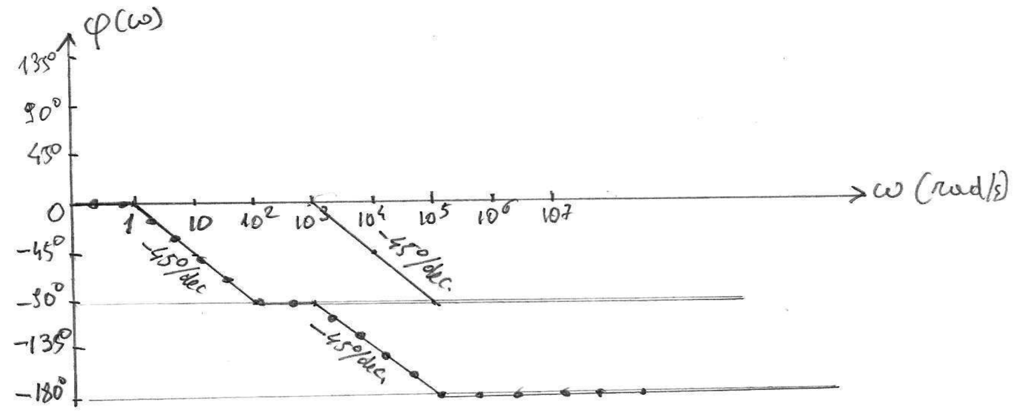
2 poli reali semplici

$\omega_{p1} = 10$ rad/s

$\omega_{p2} = 10^4$ rad/s

$20 \log_{10} 10^2 = 20 \cdot 2 = 40\,dB$





$A = 10^6$

$$F(s) = \frac{A}{(s+10)(s+1000)} = \frac{A}{(j\omega+10)(j\omega+1000)} =$$

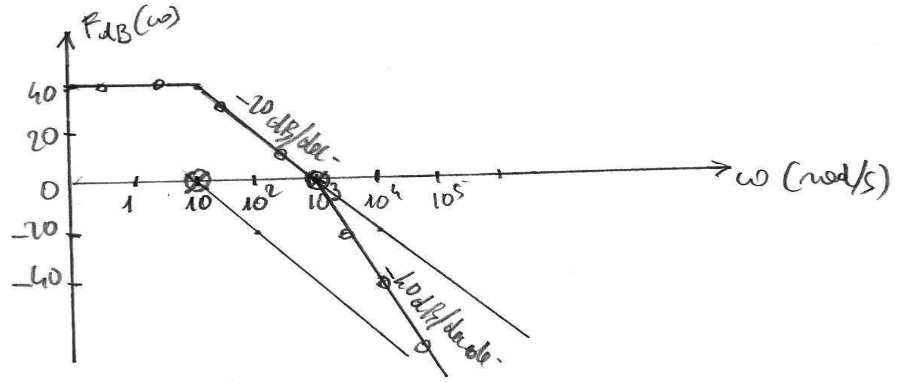
$$= \frac{10^6}{10\left(1+j\frac{\omega}{10}\right)1000\left(1+\frac{j\omega}{1000}\right)} = \frac{10^2}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{1000}\right)}$$

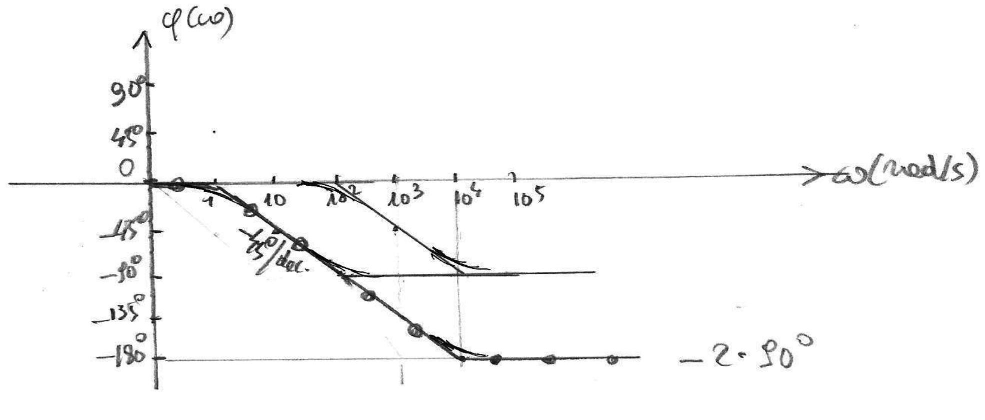
2 polo reale simplu

$\omega_{p1} = 10$ rad/s

$\omega_{p2} = 1000$ rad/s.

$20 \log_{10} 10^2 = 40$ dB



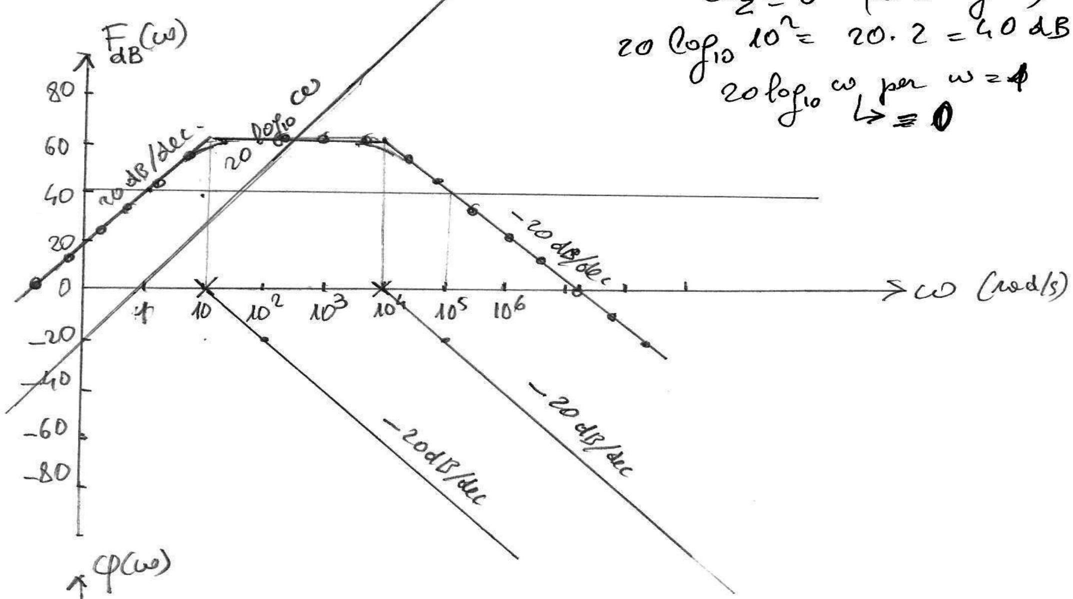


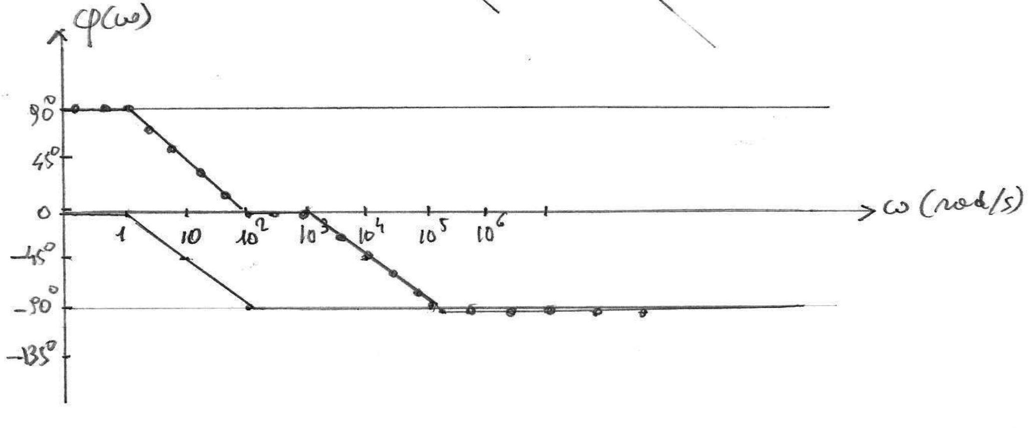
$$A = 10^7$$

$$F(s) = \frac{A\,s}{(s+10)(s+10000)} = \frac{j\omega \cdot 10^7}{10\left(\frac{s}{10}+1\right)\left(\frac{s}{10000}+1\right)10000} =$$

$$= \frac{j\omega \cdot 10^2}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^4}\right)}$$

poli reali semplici:
$\omega_{p1} = 10$ rad/s
$\omega_{p2} = 10^4$ rad/s
zero reale semplice:
$\omega_z = 0$ (nell'origine)
$20 \log_{10} 10^2 = 20 \cdot 2 = 40$ dB
$20 \log_{10} \omega$ per $\omega = 1$
$\hookrightarrow = 0$



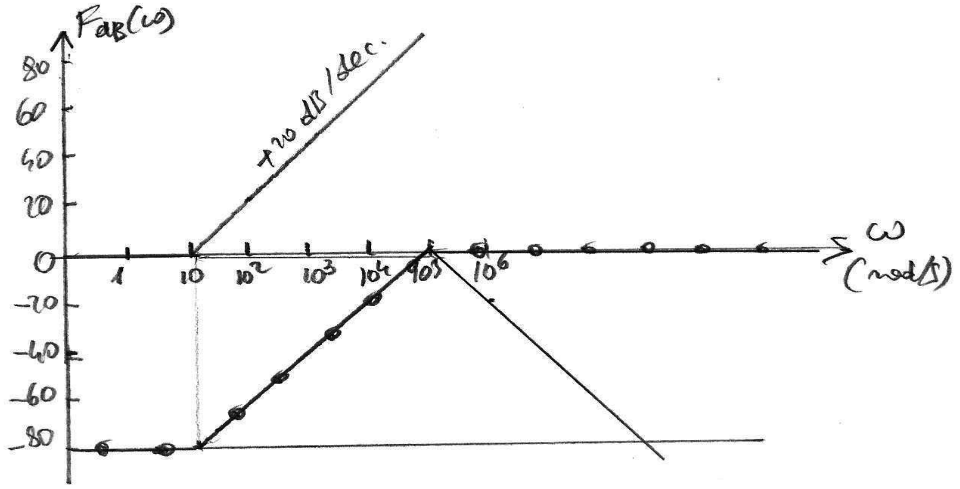


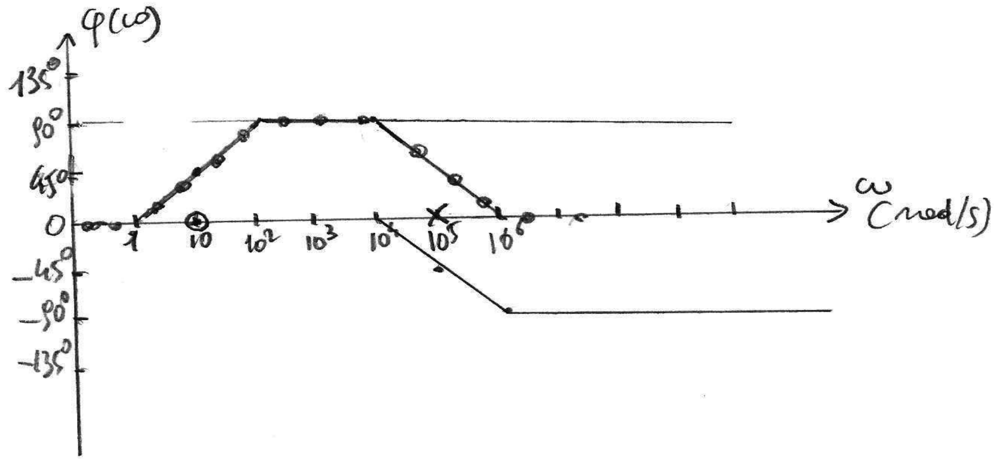
$$F(s) = \frac{10^{-4}\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{10^5}\right)}$$

zero reale sempre: $\omega_z = 10$ rad/s

polo reale sempre: $\omega_p = 10^5$ rad/s

$20\log_{10} 10^{-4} = -80$dB





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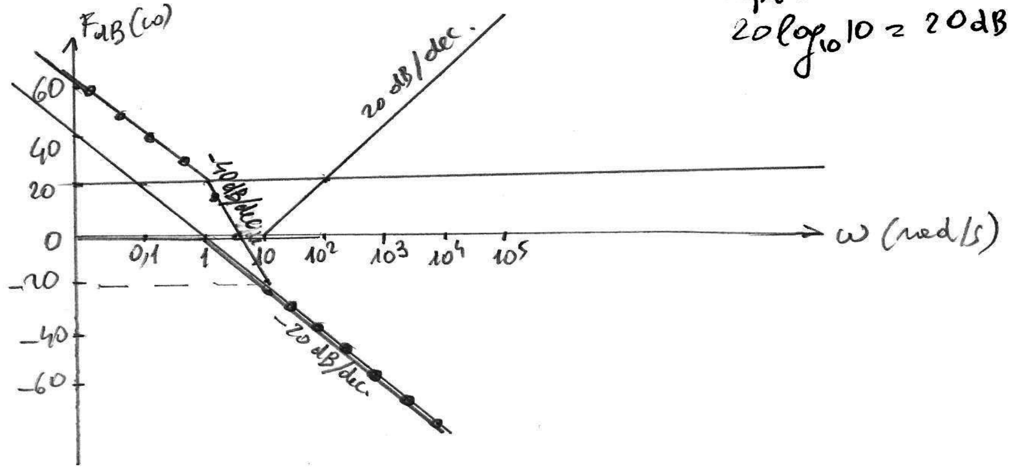
$$F(s) = \frac{s+10}{s(s+1)} \quad ; \quad F(j\omega) = \frac{j\omega+10}{j\omega(j\omega+1)} = \frac{10\left(1+\frac{j\omega}{10}\right)}{j\omega(j\omega+1)}$$

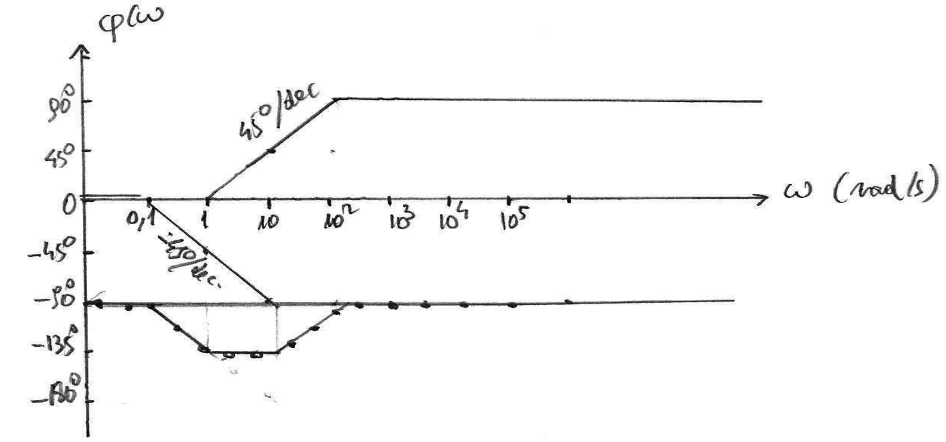
Zero reale semplice: $\omega_z = 10$ rad/s

poli reali semplici: $\omega_{p1} = 0$ (nell'origine)

$\omega_{p2} = 1$ rad/s

$20\log_{10}10 = 20\,dB$





$$F(s) = \frac{10\,(1+10s)}{(1+0{,}1s)(1+0{,}001s)^2} \qquad A = 10$$

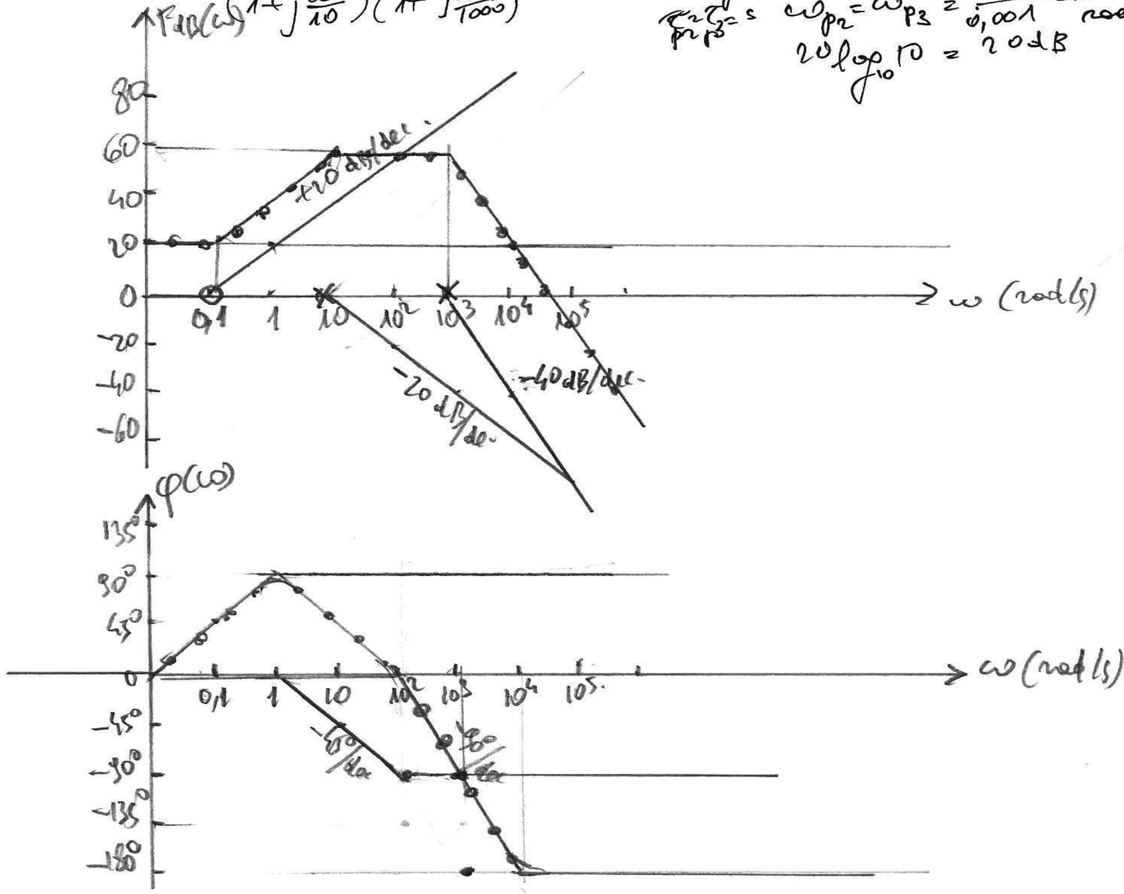
$$F(j\omega) = \frac{10\,(1+j\omega 10)}{(1+j0{,}1\omega)(1+0{,}001j\omega)^2} = \frac{10\left(1+j\frac{\omega}{0{,}1}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{1000}\right)^2}$$

1 zero reale semplice $\tau_1 = 10\,s$; $\omega_z = \frac{1}{10} = 0{,}1$ rad/s

1 polo reale semplice $\tau_{p1} = 0{,}1\,s$ $\omega_{p1} = \frac{1}{0{,}1} = 10$ rad/s

1 polo reale doppio $\tau_{p2} = \tau_{p3} = \;s$ $\omega_{p2} = \omega_{p3} = \frac{1}{0{,}001} = 1000$ rad/s

$20\log_{10} 10 = 20$ dB



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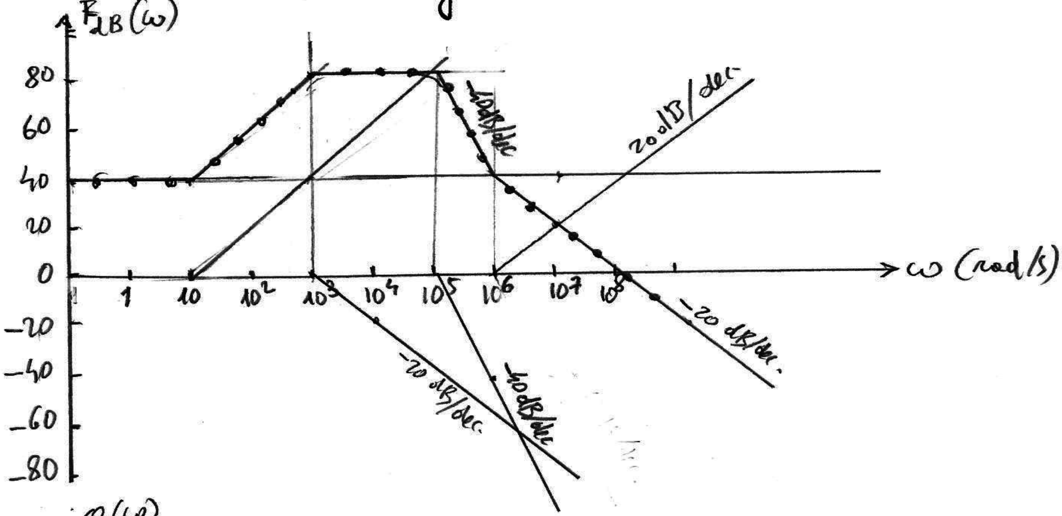
$$F(s) = \frac{100\,(1+0{,}1s)(1+10^{-6}s)}{(1+10^{-3}s)(1+10^{-5}s)^2} = \frac{100\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^6}\right)}{\left(1+\frac{j\omega}{10^3}\right)\left(1+\frac{j\omega}{10^5}\right)^2}$$

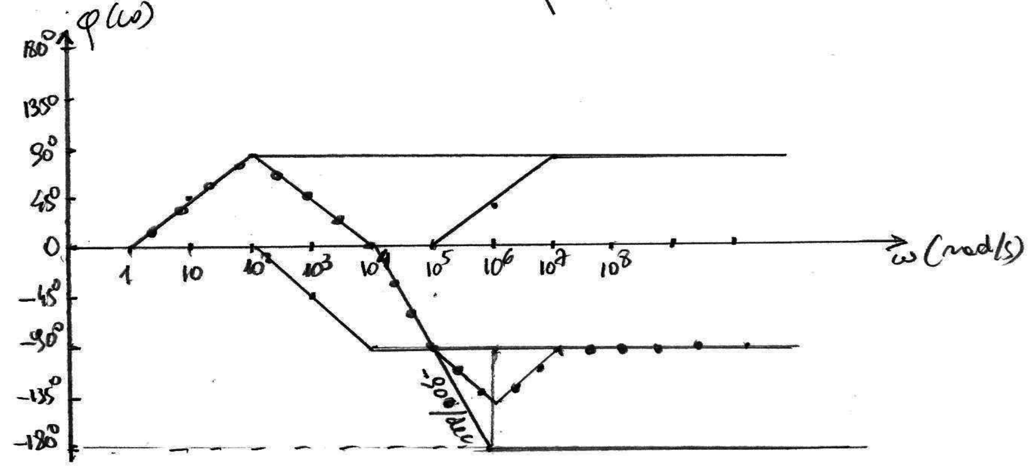
zeri reali semplici: $\omega_{z1} = 10$ rad/s; $\omega_{z2} = 10^6$ rad/s

poli reali: $\omega_{p1} = 10^3$ rad/s - semplice

$\omega_{p2} = 10^5$ rad/s - doppio

$20 \log_{10} 100 = 40$ dB





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$$F(s) = \frac{0{,}01\,(1+j\omega)}{j\omega\,(1+0{,}01\,j\omega)(1+0{,}001\,j\omega)^2} = \frac{0{,}01\,(1+j\omega)}{j\omega\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{1000}\right)^2}$$

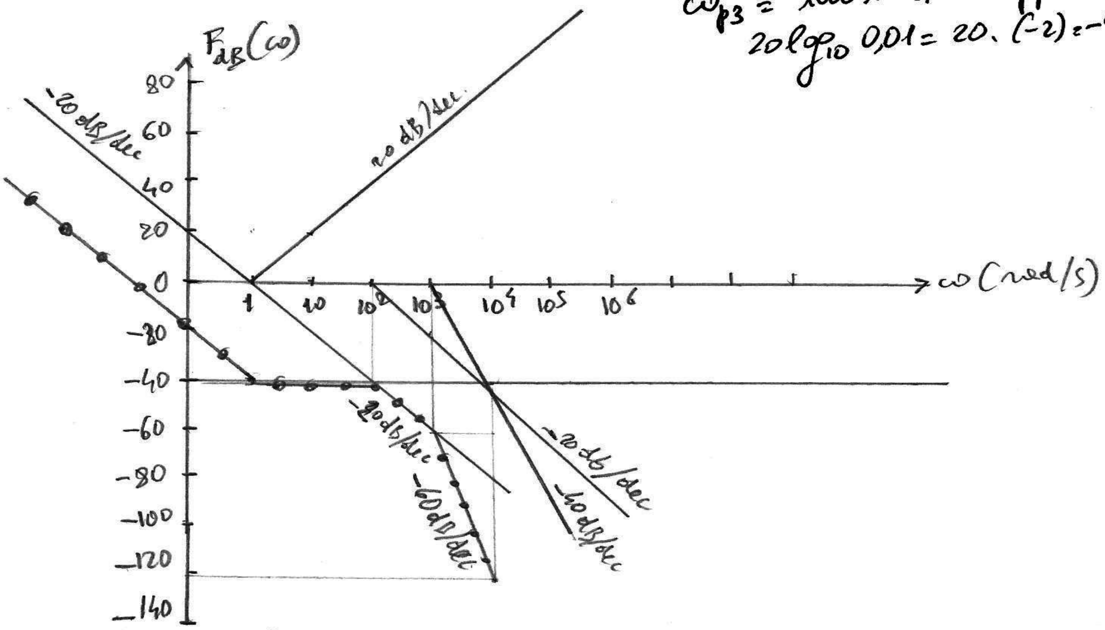
Zero reale semplice: $\omega_z = 1$ rad/s

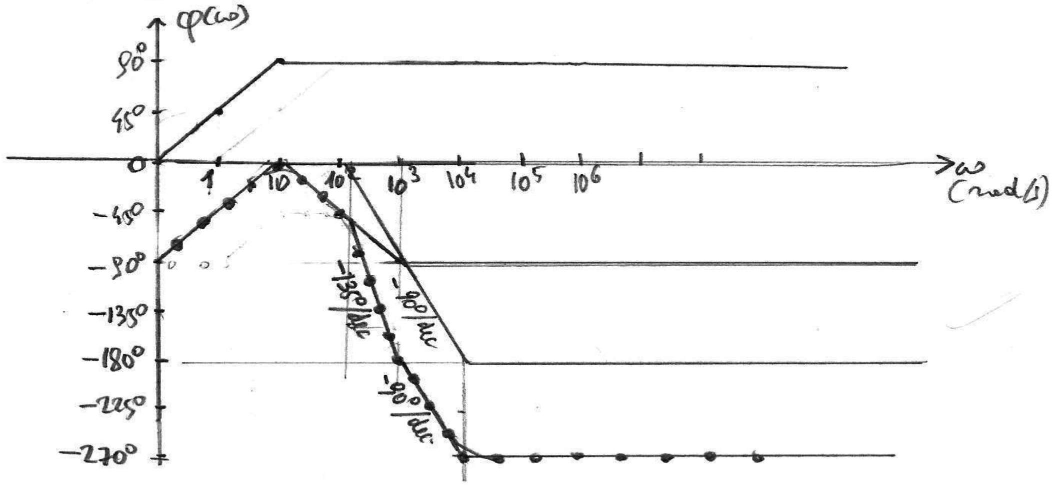
poli reali: $\omega_{p1} = 0$, nell'origine

$\omega_{p2} = 100$ rad/s - semplice

$\omega_{p3} = 1000$ rad/s - doppio

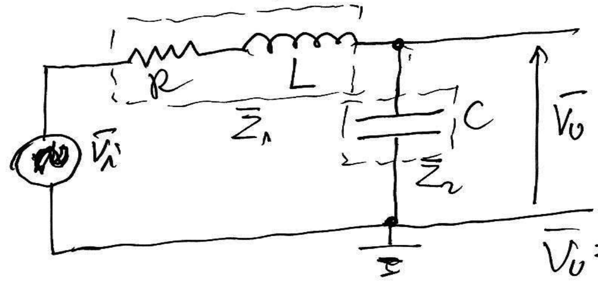
$20\log_{10} 0{,}01 = 20 \cdot (-2) = -40$ dB





Sistema del II ordine

CIRCUITO RLC SERIE



$\bar{Z}_1 = R + sL$

$\bar{Z}_2 = \frac{1}{sC}$

$$\bar{V}_0 = \frac{\bar{V}_i \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{\bar{V}_i \frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$F(s) = \frac{\bar{V}_0}{\bar{V}_i} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{\cancel{\frac{1}{sC}}}{\frac{RCs + s^2LC + 1}{\cancel{sC}}} = \frac{1}{RCs + s^2LC + 1} =$$

$$= \frac{1}{j\omega RC - \omega^2 LC + 1}$$

DEFINIZIONI: 1) PULSAZIONE NATURALE:

$\omega_n = \frac{1}{\sqrt{LC}}$; $LC = \frac{1}{\omega_n^2}$

2) COEFFICIENTE (O FATTORE) DI SMORZAMENTO ξ

SI PONE $RC = \frac{2\xi}{\omega_n}$

$$\xi = \frac{\omega_n RC}{2} = \frac{1}{\sqrt{LC}} \cdot \frac{RC}{2} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

ξ è $\propto$ a R, FISSATI L e C

$$F(j\omega) = \frac{1}{j\omega \cdot \frac{2\xi}{\omega_n} - \frac{\omega^2}{\omega_n^2} + 1} =$$

$$= \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\frac{2\xi\omega}{\omega_n}};$$

$$|F(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

$$\varphi(\omega) = -\operatorname{arctg} \frac{\frac{2\xi\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2}}$$

Calcolo dei poli di F(s)

$$F(s) = \frac{1}{RCs + s^2LC + 1} = \frac{1}{\frac{2\zeta s}{\omega_n} + \frac{s^2}{\omega_n^2} + 1} =$$

$$\boxed{\begin{aligned} RC &= \frac{2\zeta}{\omega_n} \\ LC &= \frac{1}{\omega_n^2} \end{aligned}} \qquad = \frac{\omega_n^2}{s^2 + 2\zeta s\omega_n + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2} =$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} =$$

$$= -\omega_n\left(\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

Se $\zeta > 1$ i poli sono reali e distinti.

Se $\zeta = 1$ i poli sono reali e coincidenti.

Se $\zeta < 1$ i poli sono complessi coniugati.

Per $\zeta = 1$ si ha lo smorzamento critico

Caso dei poli complessi coniugati ($\zeta < 1$)

Derivando rispetto a ω il modulo $|F(j\omega)|$ ed uguagliando a zero, si ottiene il valore della pulsazione di risonanza ω_R, per cui $|F(j\omega)|$ è massimo.

$$|F(j\omega)| = \frac{1}{D(\omega)}, \quad \text{dove } D(\omega) = \sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\omega z}{\omega_n}\right)^2}$$

$$\frac{d}{d\omega} D^2(\omega) = \frac{d}{d\omega}\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\omega z}{\omega_n}\right)^2\right] =$$

$$= 2\left(1-\frac{\omega^2}{\omega_n^2}\right)\cdot\left(-\frac{2\omega}{\omega_n^2}\right) + 2\left(\frac{2\omega z}{\omega_n}\right)\cdot\frac{2z}{\omega_n}$$

$$\frac{d}{d\omega} D^2(\omega) = 0$$

problema di minimo ($|F(j\omega)|$ è massimo se $D(\omega)$ è minimo)

$$-4\frac{\omega}{\omega_n^2} + \frac{4\omega^3}{\omega_n^4} + \frac{8\omega z^2}{\omega_n^2} = 0$$

$$-1 + \frac{\omega^2}{\omega_n^2} + \cancel{8}2 z^2 = 0 \; ; \quad -\omega_n^2 + \omega^2 + \cancel{8}2 z^2\omega_n^2 = 0$$

$$\omega^2 = \omega_n^2 - \cancel{8}2 z^2\omega_n^2 =$$

$$= \omega_n^2(1-2z^2)$$

$$\omega = \omega_R = \omega_n\sqrt{1-2z^2} \leftarrow$$ PULSAZIONE DI RISONANZA ω_R

Per $\omega = \omega_R = \omega_n\sqrt{1-2z^2}$

$$|F(j\omega)|_{MAX} = \frac{1}{\sqrt{\left[1-\frac{\cancel{\omega_n^2}(1-2z^2)}{\cancel{\omega_n^2}}\right]^2 + \frac{4z^2}{\cancel{\omega_n^2}}\cdot\cancel{\omega_n^2}(1-2z^2)}} =$$

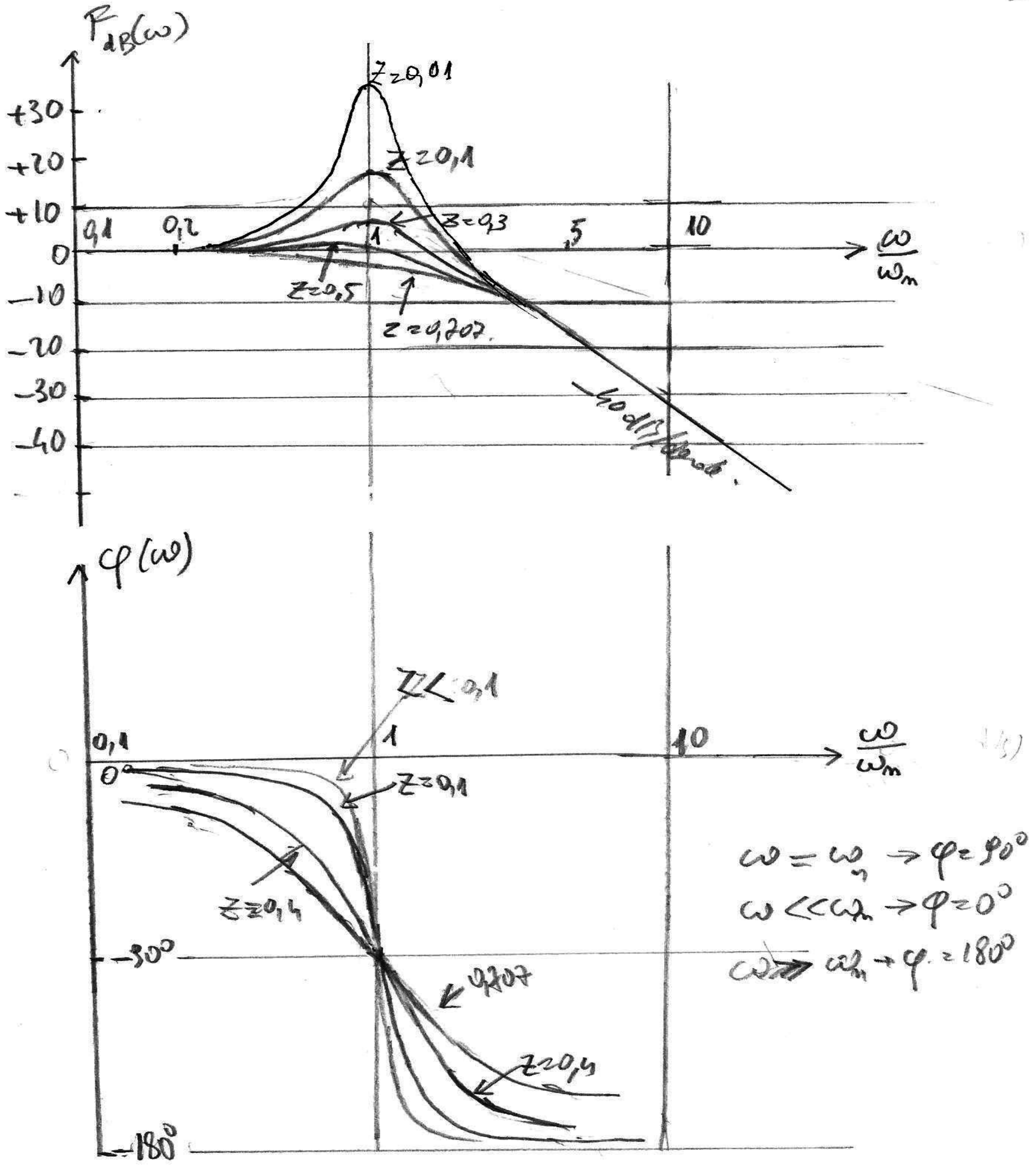
$$= \frac{1}{\sqrt{[\cancel{1}-\cancel{1}+2z^2]^2 + 4z^2 - 8z^4}} = \frac{1}{\sqrt{4z^4 + 4z^2 - 8z^4}} =$$

$$|F(j\omega)|_{MAX} = \frac{1}{2z\sqrt{1-z^2}}$$

Poli complessi coniugati.

VALORI DI $|F(j\omega)|$ PER $\omega = \omega_n$

ζ	$\lvert F(j\omega_n)\rvert = \dfrac{1}{2\zeta}$
1	$\frac{1}{2\cdot 1} = 0{,}5 \rightarrow 20\log_{10} 0{,}5 = -6\ dB$ ← smorzamento critico
0,707	$\frac{1}{2\cdot 0{,}707} = \frac{1}{1{,}414} = \frac{1}{\sqrt{2}} = 0{,}707 \rightarrow 20\log_{10} 0{,}707 = -3\ dB$ ← smorzamento ~~critico~~ prossimo a quello critico
0,5	$\frac{1}{2\cdot 0{,}5} = 1 \longrightarrow 20\log_{10} 1 = 0\ dB$
0,25	$\frac{1}{2\cdot 0{,}25} = \frac{1}{0{,}5} = 2 \longrightarrow 20\log_{10} 2 = 20\cdot 0{,}3 = 6\ dB$
0,2	$\frac{1}{2\cdot 0{,}2} = \frac{1}{0{,}4} = 2{,}5 \rightarrow 20\log_{10} 2{,}5 = 7{,}958\ dB$
0,1	$\frac{1}{2\cdot 0{,}1} = \frac{1}{0{,}2} = 5 \rightarrow 20\log_{10} 5 = 13{,}97\ dB$
0,05	$\frac{1}{2\cdot 0{,}05} = \frac{1}{0{,}1} = 10 \rightarrow 20\log_{10} 10 = 20\ dB$
0,01	$\frac{1}{2\cdot 0{,}01} = \frac{1}{0{,}02} = 50 \rightarrow 20\log_{10} 50 = 33{,}97\ dB$



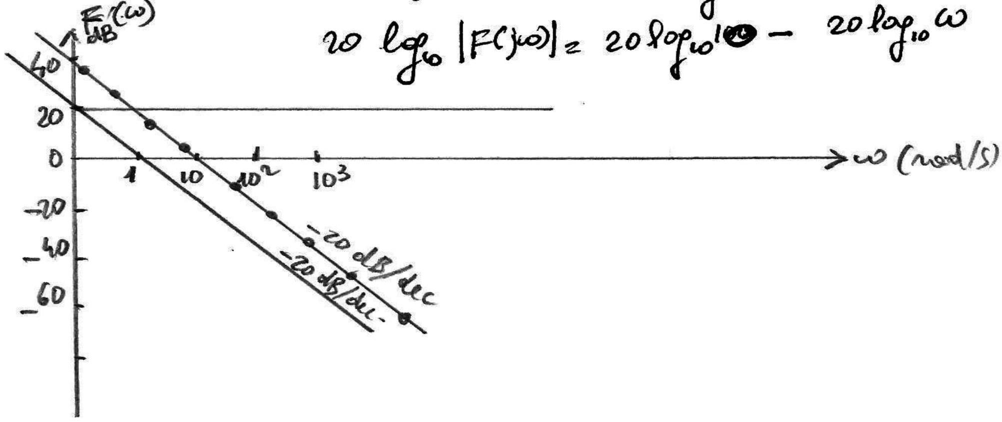
$F_{dB}(\omega)$
+30
+20
+10
0
−10
−20
−30
−40
0,1
0,2
1
5
10
$\frac{\omega}{\omega_n}$
z=0,01
z=0,1
z=0,3
z=0,5
z=0,707
−40 dB/dekad.
$\varphi(\omega)$
0,1
1
10
$\frac{\omega}{\omega_n}$
0°
−90°
−180°
z<<0,1
z=0,1
z=0,4
0,707
z=0,4
$\omega = \omega_n \to \varphi = 90^0$
$\omega << \omega_n \to \varphi = 0^0$
$\omega >> \omega_n \to \varphi = 180^0$

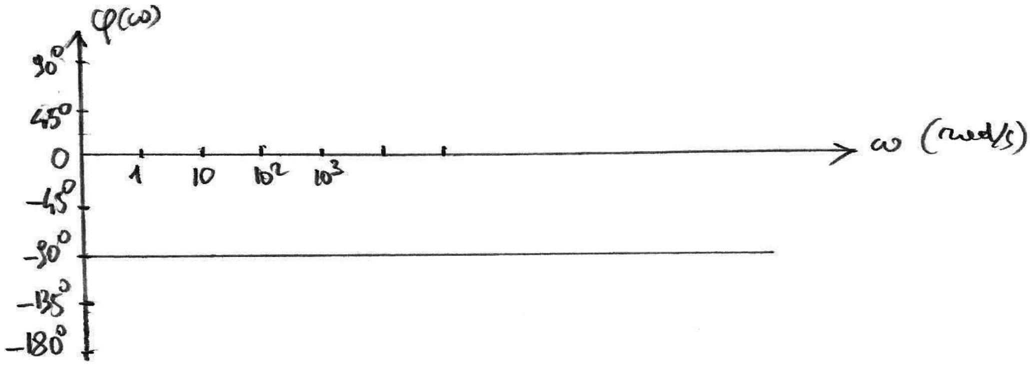
$$F(j\omega) = \frac{100}{s} = \frac{100}{j\omega}$$

1 polo reale semplice $\omega_z = 0$ 14

$20 \log_{10} 100 = 40 \text{ dB}$

$$20 \log_{10} |F(j\omega)| = 20 \log_{10} 100 - 20 \log_{10} \omega$$



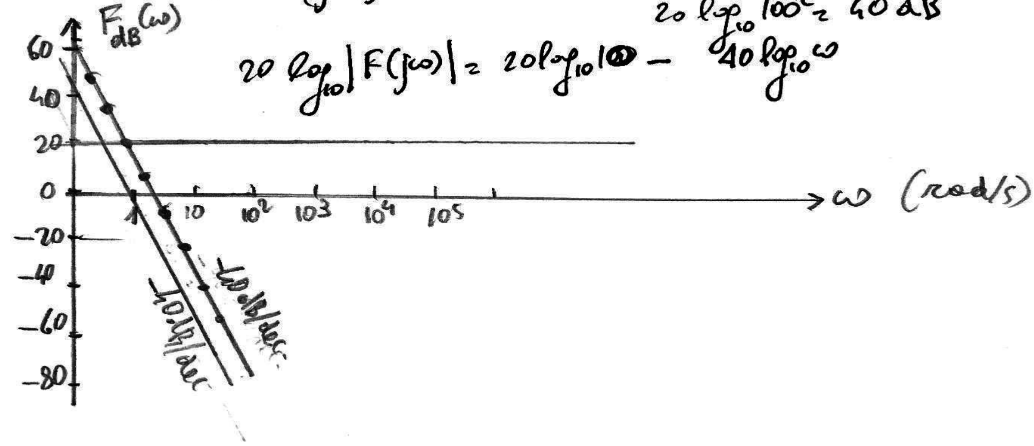


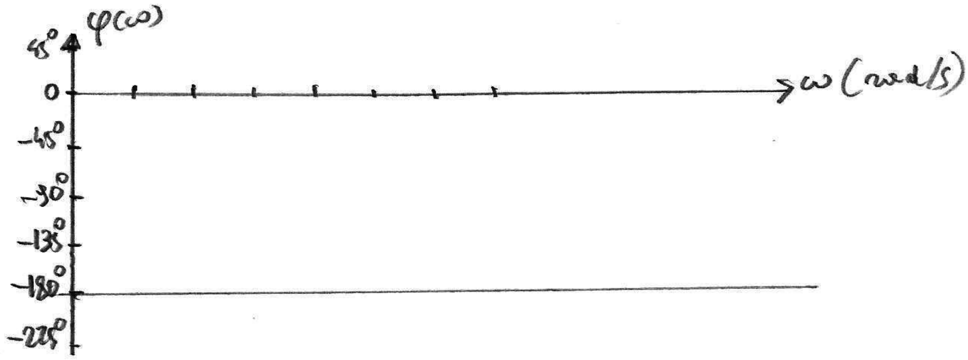
$$F(j\omega) = \frac{100}{s^2} = \frac{100}{(j\omega)^2} =$$

1 polo reale doppio $\omega_2 = 0$ 15

$20\log_{10} 100^2 = 40\,dB$

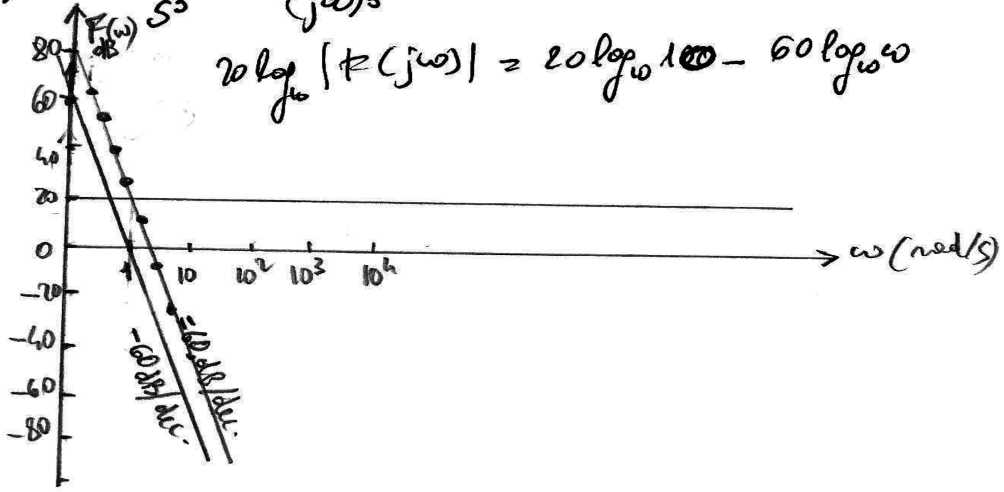
$20\log_{10}|F(j\omega)| = 20\log_{10}100 - 40\log_{10}\omega$

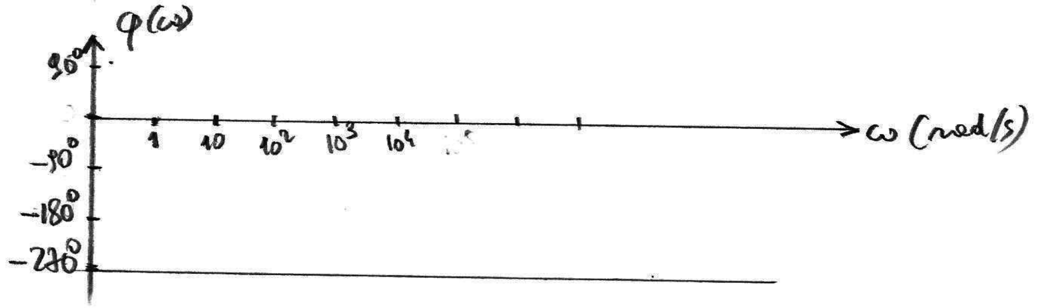




$F(j\omega) = \frac{100}{s^3} = \frac{100}{(j\omega)^3}$ 1 polo reale triplo per $\omega_2 = 0$ 16

$20\log_{10}|F(j\omega)| = 20\log_{10}100 - 60\log_{10}\omega$





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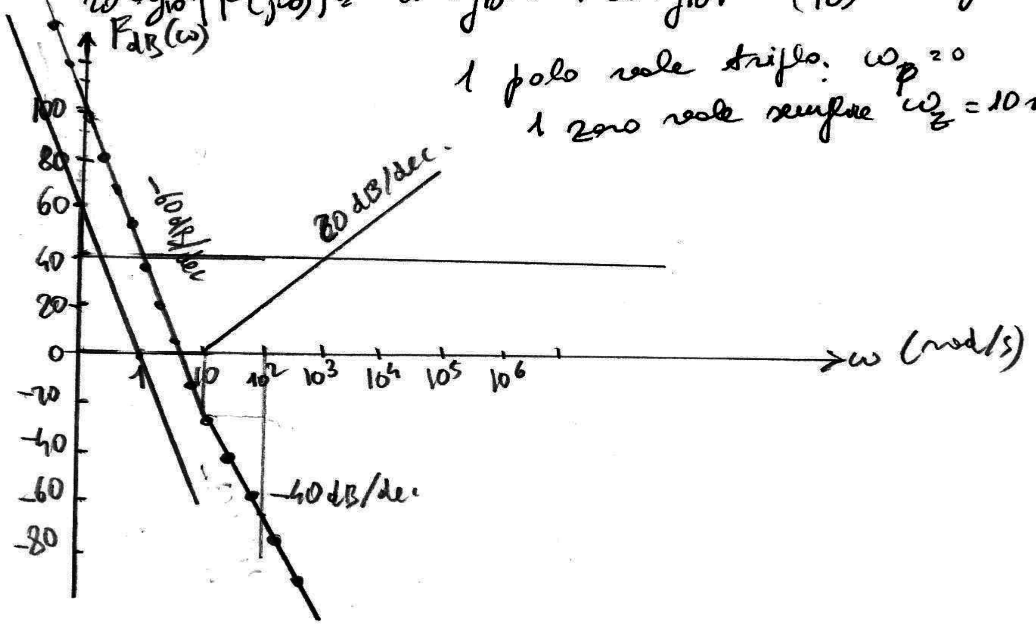
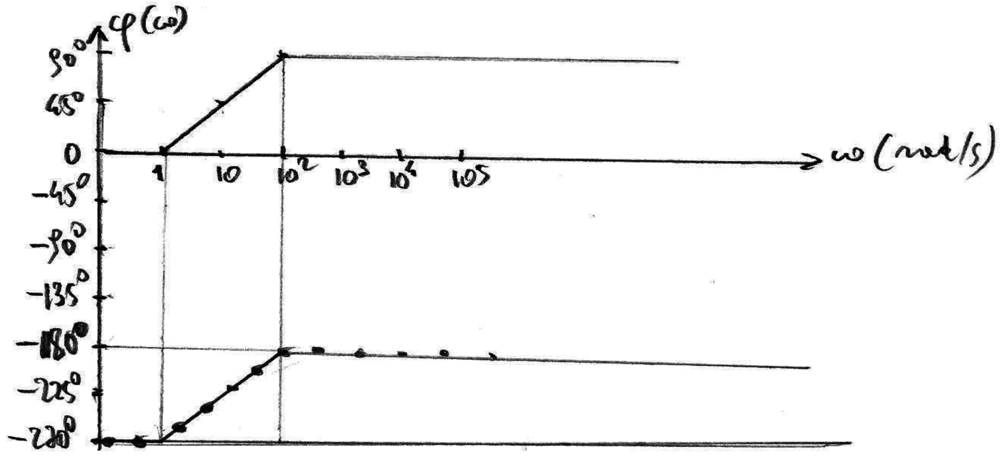
$$F(j\omega) = \frac{100\left(1+\frac{s}{10}\right)}{s^3} = \frac{100\left(1+\frac{j\omega}{10}\right)}{(j\omega)^3}$$

$$20\log_{10}|F(j\omega)| = 20\log_{10}100 + 20\log_{10}\sqrt{1+\left(\frac{\omega}{10}\right)^2} - 60\log_{10}\omega$$

1 polo reale triplo: $\omega_p = 0$

1 zero reale semplice $\omega_z = 10$ rad/s





$$F(j\omega) = \frac{100\left(1+\frac{s}{10}\right)\left(1+\frac{s}{1000}\right)}{s^3} = \frac{100\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{10^3}\right)}{(j\omega)^3}$$

$$20\log_{10}|F(j\omega)| = 20\log_{10}100 + 20\log_{10}\sqrt{1+\left(\frac{\omega}{10}\right)^2} + 20\log_{10}\sqrt{1+\left(\frac{\omega}{10^3}\right)^2} - 60\log_{10}\omega$$

1 polo reale triplo: $\omega_p = 0$

2 zeri reali semplici: $\omega_{z1} = 10$ rad/s, $\omega_{z2} = 10^3$ rad/s