DIAGRAMMI DI NYQUIST

(PASSO BASSO) 1 polo : $\omega_p = 10$ rad/s

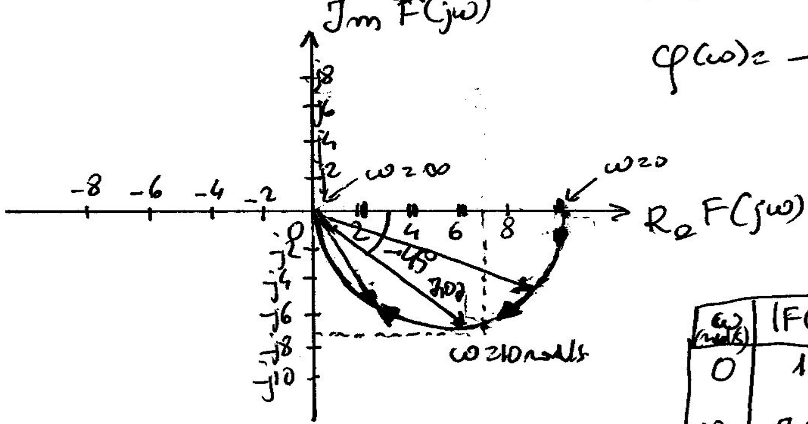
1) $$F(s) = \frac{K}{1+\tau s} = \frac{K}{1+j\omega\tau}$$

Es. $$F(s) = \frac{100}{s+10} = \frac{100}{10\left(\frac{s}{10}+1\right)} =$$

$\tau = 0{,}1$ s $$= \frac{10}{(1+0{,}1s)} = \frac{10}{(1+0{,}1j\omega)}$$

$$|F(j\omega)| = \frac{K}{\sqrt{1+(\omega\tau)^2}}$$

$$\varphi(\omega) = -\operatorname{arctg}(\omega\tau)$$



ω (rad/s)	|F(jω)|	φ(ω)
0	10	0°
10	7,07	−45°
∞	0	−90°

$$|F(j\omega)| = \frac{10}{\sqrt{1+(0{,}1\omega)^2}}$$

$\omega = 10$ rad

per $\omega = 10$ rad/s

$$|F(j\omega)| = \frac{10}{\sqrt{1+1}} = \frac{10}{\sqrt{2}} = 7{,}07$$

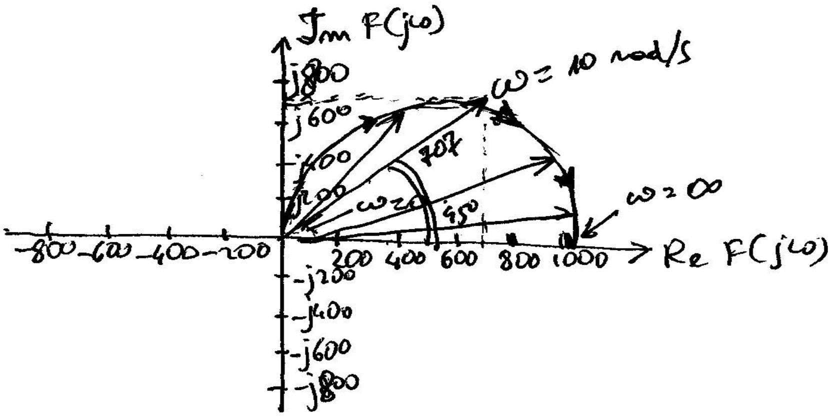
$$\varphi(\omega) = -\operatorname{arctg}(0{,}1\omega) = -\operatorname{arctg} 1 = -45°$$

2) (PASSA ALTO) 1 polo: $\omega_p = 10$ rad/s ; 1 zero: $\omega_z = 0$

$$F(s) = \frac{K\tau s}{1+\tau s} = \frac{K j\omega\tau}{1 + j\omega\tau} = \frac{K\tau s}{1+\tau s}$$

$\tau = 0{,}1\,s.$

ES: $$F(s) = \frac{10^3\, j\omega \cdot 0{,}1}{1 + j\omega \cdot 0{,}1} = \frac{j\omega 100}{1 + j\omega \cdot 0{,}1}$$



$$|F(j\omega)| = \frac{100\,\omega}{\sqrt{1+(0{,}1\omega)^2}}$$

$$\varphi(\omega) = \frac{\pi}{2} - \operatorname{arctg}(0{,}1\omega) = 90^\circ - \operatorname{arctg}(0{,}1\omega)$$

per $\omega = 10$ rad/s $|F(j\omega)| = \dfrac{100 \cdot 10}{\sqrt{1+1}} = \dfrac{1000}{\sqrt{2}} = 707$

$\varphi(\omega) = 90^\circ - 45^\circ = 45^\circ$

$\lim_{\omega \to \infty} |F(j\omega)| = \dfrac{j\omega 100}{j\omega \cdot 0{,}1} = 1000$

$\varphi(\omega) = 90^\circ - 90^\circ = 0$

ω (rad/s)	|F(jω)|	φ(ω)
0	0	90°
10	707	45°
∞	1000	0°

3) PASSA BANDO

2 poli : $\omega_{p1} = 10$ rad/s , $\omega_{p2} = 1000$ rad/s

1 zero : $\omega_z = 0$ $\qquad K = 100$

$$F(s) = \frac{Ks}{(1+\tau_1 s)(1+\tau_2 s)} = \frac{j\omega K}{(1+j\omega\tau_1)(1+j\omega\tau_2)}$$

$$\tau_1 = \frac{1}{\omega_{p1}} = 0{,}1\ s \quad ; \quad \tau_2 = \frac{1}{\omega_{p2}} = 0{,}001\ s.$$

$$F(s) = \frac{j\omega \cdot 100}{(1+j\omega \cdot 0{,}1)(1+j\omega\, 0{,}001)}$$

$$|F(j\omega)| = \frac{\omega \cdot 100}{\sqrt{1+(0{,}1\omega)^2}\sqrt{1+(0{,}001\omega)^2}}$$

$$\varphi(\omega) = 90^\circ - \operatorname{arctg}(0{,}1\omega) - \operatorname{arctg}(0{,}001\omega)$$

ω	$\lvert F(j\omega)\rvert$	φ
0	0	90°
10	707	45°
100	1000	0°
1000	707	-45°
∞	0	-90°

$$\lim_{\omega\to\infty} |F(j\omega)| = \frac{\omega \cdot 100}{\omega \cdot 0{,}1 \cdot \omega \cdot 0{,}001} = 0$$

$$\varphi(\omega) = 90^\circ - 90^\circ - 90^\circ = -90^\circ$$

per $\omega = 10$ rad/s

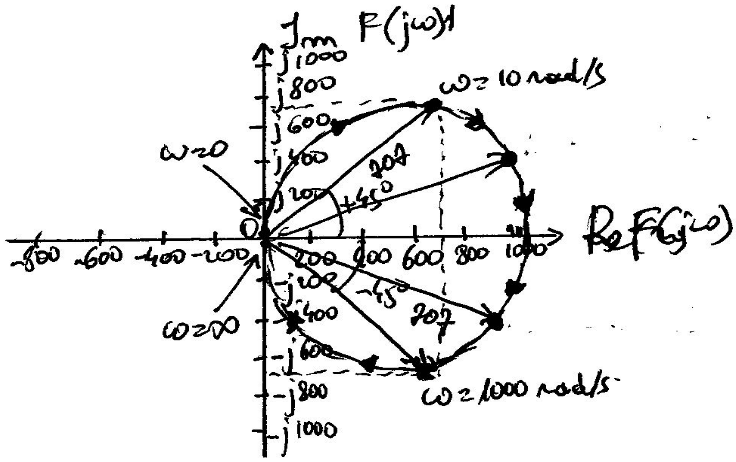
$$\varphi(\omega) = 90^\circ - \operatorname{arctg} 1 - \cancel{\operatorname{arctg} 0{,}01} \simeq 45^\circ$$

$$|F(j\omega)| = \frac{10 \cdot 100}{\sqrt{1+1} \cdot \sqrt{1+(0{,}001 \cdot 10)^2}} = \frac{1000}{\sqrt{2} \cdot \sqrt{1+(0{,}01)^2}} \simeq \frac{1000}{\sqrt{2}} = 707$$

$\omega = 1000$ rad/s

$$|F(j\omega)| = \frac{1000 \cdot 100}{\sqrt{1+(0{,}1 \cdot 10^3)^2}\sqrt{1+(0{,}001 \cdot 10^3)^2}} = \frac{10^5}{\sqrt{1+100^2}\sqrt{2}} \simeq \frac{10^5}{100\sqrt{2}} = \frac{10^3}{\sqrt{2}} = 707$$

$$\varphi(\omega) = 90^\circ - \operatorname{arctg}(0{,}1 \cdot 1000) - \operatorname{arctg} 1 = 90^\circ - 90^\circ - 45^\circ = -45^\circ$$



for $\omega = 100$ rad/s.

$$|F(j\omega)| = \frac{100 \cdot 100}{\sqrt{1+(0{,}1\cdot 100)^2}\sqrt{1+(0{,}001\cdot 100)^2}} \approx \frac{10^4}{\sqrt{10^2}\cdot\sqrt{1{,}001}} \approx \frac{10^4}{10} = 1000$$

$$\varphi(\omega) = 90^\circ - \operatorname{arctg}(0{,}1\cdot 100) - \operatorname{arctg}(0{,}001\cdot 100)$$

$$\approx 90^\circ - 90^\circ - 0^\circ = 0^\circ$$

4) passa basso 2° ordine.

2 poli: $\omega_{p1} = 10$ rad/s $\quad \omega_{p2} = 1000$ rad/s.

$\tau_1 = \frac{1}{\omega_{p1}} = 0{,}1\,s.$; $\tau_2 = \frac{1}{\omega_{p2}} = 0{,}001\,s$ | $K = 100$

$$F(s) = \frac{K}{(1+\tau_1 s)(1+\tau_2 s)} = \frac{\cancel{1000}\;100}{(1+0{,}1j\omega)(1+0{,}001j\omega)}$$

$$|F(j\omega)| = \frac{100}{|(1+0{,}1j\omega)(1+0{,}001j\omega)|} = \frac{100}{\sqrt{1+(0{,}1\omega)^2}\sqrt{1+(0{,}001\omega)^2}}$$

$$\varphi(\omega) = -\operatorname{arctg}(0{,}1\omega) - \operatorname{arctg}(0{,}001\omega)$$

ω	$\lvert F(j\omega)\rvert$	φ
0	100	0
10	70,7	$-45°$
1000	0,707	$-135°$
∞	0	$-180°$

per $\omega = 10$ rad/s

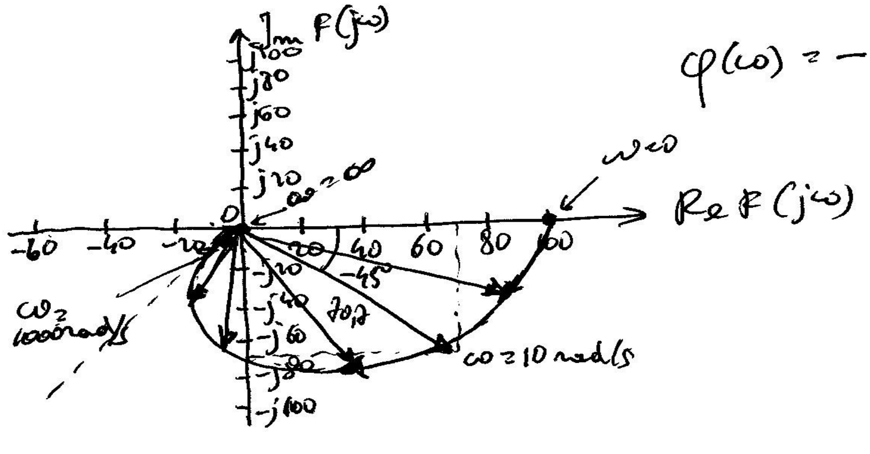
$$|F(j\omega)| = \frac{100}{\sqrt{2}\cdot\sqrt{1}} = 70{,}7$$

$$\varphi(\omega) = -45°$$

per $\omega = 1000$ rad/s

$$|F(j\omega)| = \frac{100}{\sqrt{100^2}\cdot\sqrt{2}} = \frac{1}{\sqrt{2}} = 0{,}707.$$

$$\varphi(\omega) = -90° - 45° = -135°$$



PASSA BASSO 4° ordine.

$$F(s) = \frac{K}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)(1+\tau_4 s)}$$

$K = 100$

ES. $\tau_1 = 0{,}1\,s$, $\omega_{p1} = 10$ rad/s.
$\tau_2 = 1\,s$, $\omega_{p2} = 1$ rad/s.
$\tau_3 = 10\,s$, $\omega_{p3} = 0{,}1$ rad/s.
$\tau_4 = 0{,}01\,s$, $\omega_{p4} = 100$ rad/s.

$$F(s) = \frac{100}{(1+0{,}1 j\omega)(1+j\omega)(1+10j\omega)(1+0{,}01 j\omega)}$$

ω	$\lvert F(j\omega)\rvert$	$\varphi(\omega)$
0	100	0
∞	0	$-360°$
0,1	70,7	$-45°$
1	7,07	$-135°$
10	0,707	$-225°$
100	$\simeq 0$	$-315°$

$\varphi(\omega) = -\operatorname{arctg} 0{,}1\omega - \operatorname{arctg} \omega - \operatorname{arctg} 10\omega - \operatorname{arctg} 0{,}01\omega$

$$\lvert F(j\omega)\rvert_{\omega = 0{,}1 \text{ rad/s}} \simeq \frac{100}{\sqrt{1+(10\cdot 0{,}1)^2}} = \frac{100}{\sqrt{2}} = 70{,}7$$

$\varphi(\omega) \simeq -\operatorname{arctg} 10 \cdot 0{,}1 \simeq -45°$

$$\lvert F(j\omega)\rvert_{\omega = 1 \text{ rad/s}} \simeq \frac{100}{\sqrt{1+\cancel{0{,}1}^2}\sqrt{2}\sqrt{1+10^2}\sqrt{1+0{,}01^2}} \simeq \frac{100}{\sqrt{2}\cdot 10} \simeq \frac{10}{\sqrt{2}} = 7{,}07$$

$\varphi(\omega) = -\cancel{\operatorname{arctg} 0{,}1} - \operatorname{arctg} 1 - \operatorname{arctg} 10 - \cancel{\operatorname{arctg} 0{,}01} \simeq -45° - 90° = -135°$

$$\lvert F(j\omega)\rvert_{\omega = 10 \text{ rad/s}} \simeq \frac{100}{\sqrt{1+1}\sqrt{1+10^2}\sqrt{1+100^2}\sqrt{1+0{,}1^2}} \simeq \frac{100}{\sqrt{2}\cdot 10\cdot 10} = \frac{1}{\sqrt{2}} = 0{,}707$$

$\varphi(\omega) = -\operatorname{arctg} 1 - \operatorname{arctg} 10 - \operatorname{arctg} 100 - \cancel{\operatorname{arctg} 0{,}1} = -45° - 90° - 90° = -225°$

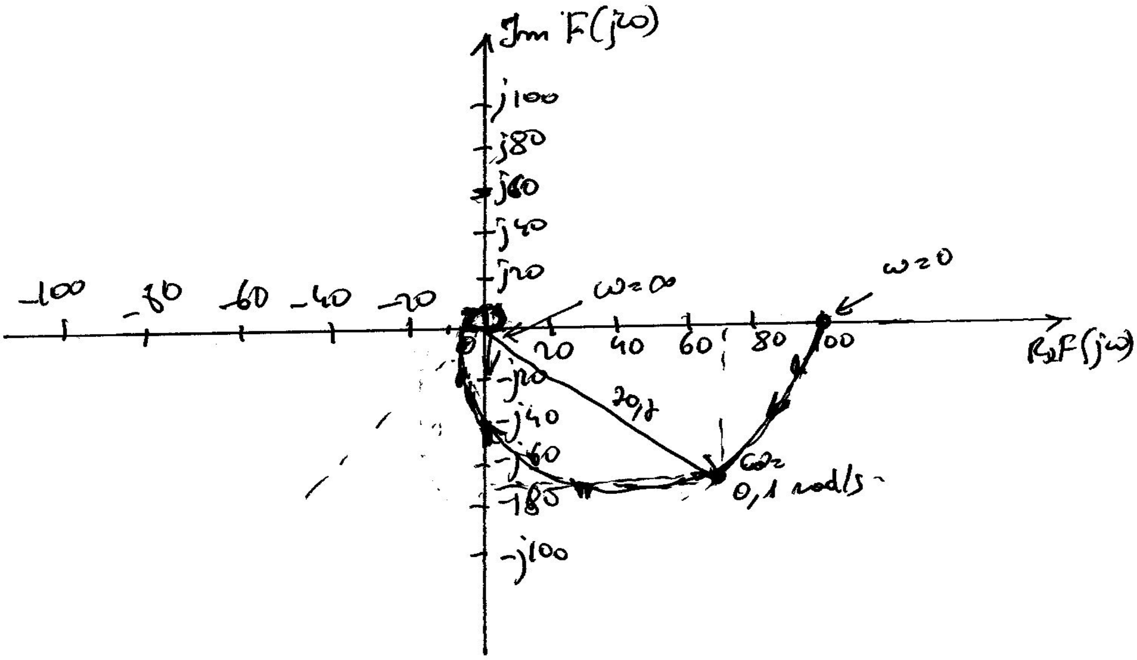
$\omega = 100$ rad/s.

$$|F(j\omega)| = \frac{100}{\sqrt{\cancel{1}+10^2}\sqrt{\cancel{1}+100^2}\sqrt{\cancel{1}+1000^2}\sqrt{1+1}} \approx$$

$$\approx \frac{100}{10\cdot 10\cdot 1000\cdot \sqrt{2}} = \frac{1}{1000\sqrt{2}} = \frac{0{,}707}{1000} = 0{,}00707 \approx 0$$

$$\varphi(\omega) = -\text{arctg}\,10 - \text{arctg}\,100 - \text{arctg}\,1000 - \text{arctg}\,1 =$$

$$= -90^\circ - 90^\circ - 90^\circ - 45^\circ = -315^\circ$$

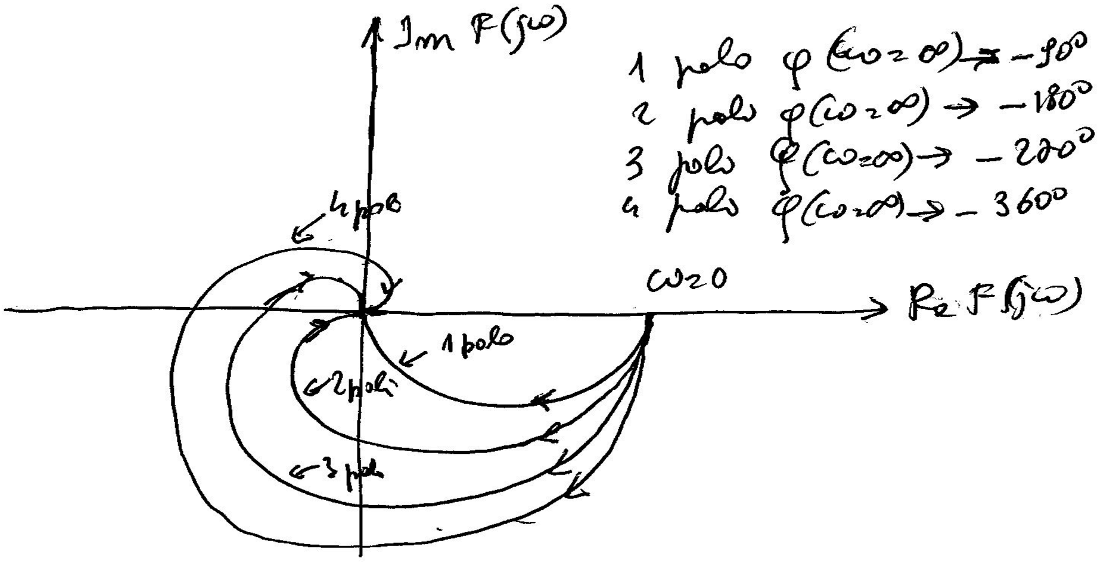


1 polo $\varphi(\omega = \infty) \rightarrow -90^\circ$

2 poli $\varphi(\omega = \infty) \rightarrow -180^\circ$

3 poli $\varphi(\omega = \infty) \rightarrow -270^\circ$

4 poli $\varphi(\omega = \infty) \rightarrow -360^\circ$



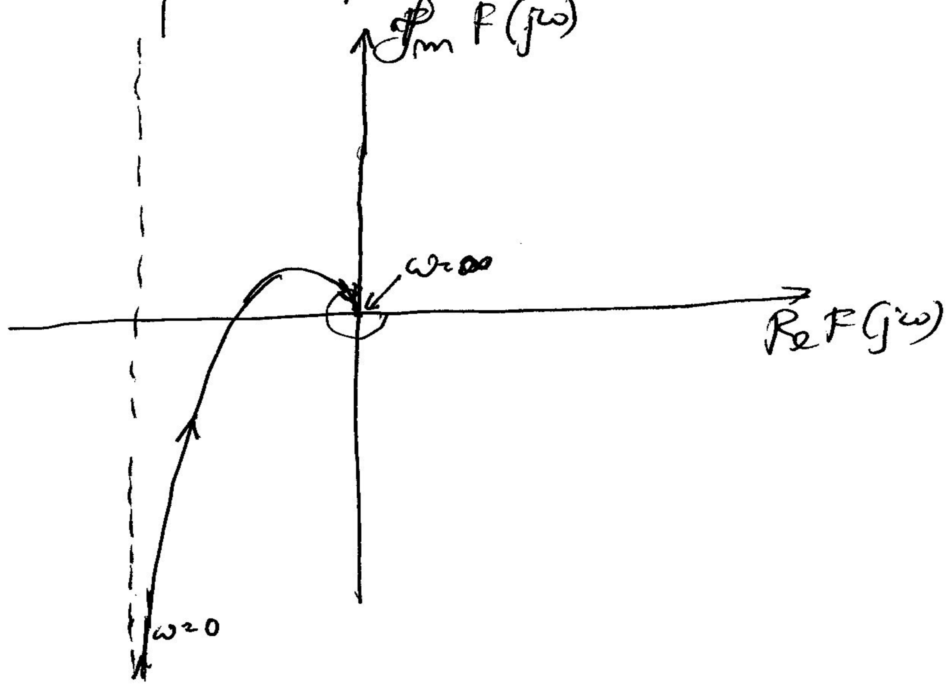
$$F(s) = \frac{K}{s(1+\tau_1 s)(1+\tau_2 s)} = \frac{K}{j\omega(1+j\omega\tau_1)(1+j\omega\tau_2)}$$

2 polo reali semplici
1 polo nullo

$$|F(j\omega)| = \frac{K}{\omega\sqrt{1+(\omega\tau_1)^2}\sqrt{1+(\omega\tau_2)^2}}$$

$$\varphi(\omega) = -90^\circ - \arctg\,\omega\tau_1 - \arctg\,\omega\tau_2$$

ω	$\lvert F(j\omega)\rvert$	φ
0	∞	-90°
∞	0	-270°



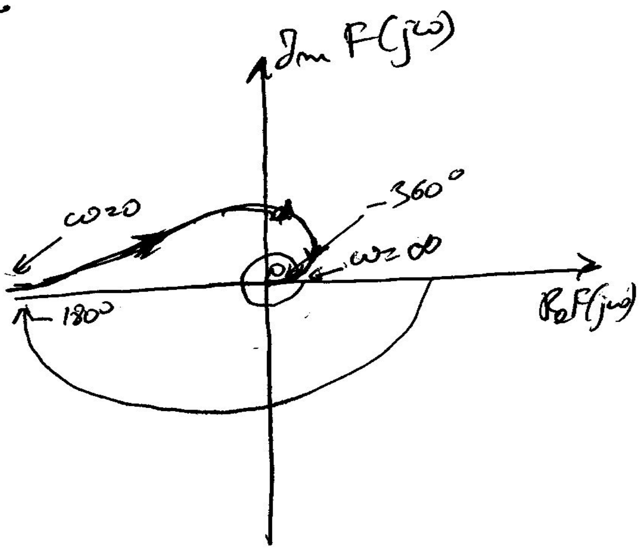
$$F(s) = \frac{K}{s^2(1+\tau_1 s)(1+\tau_2 s)}$$

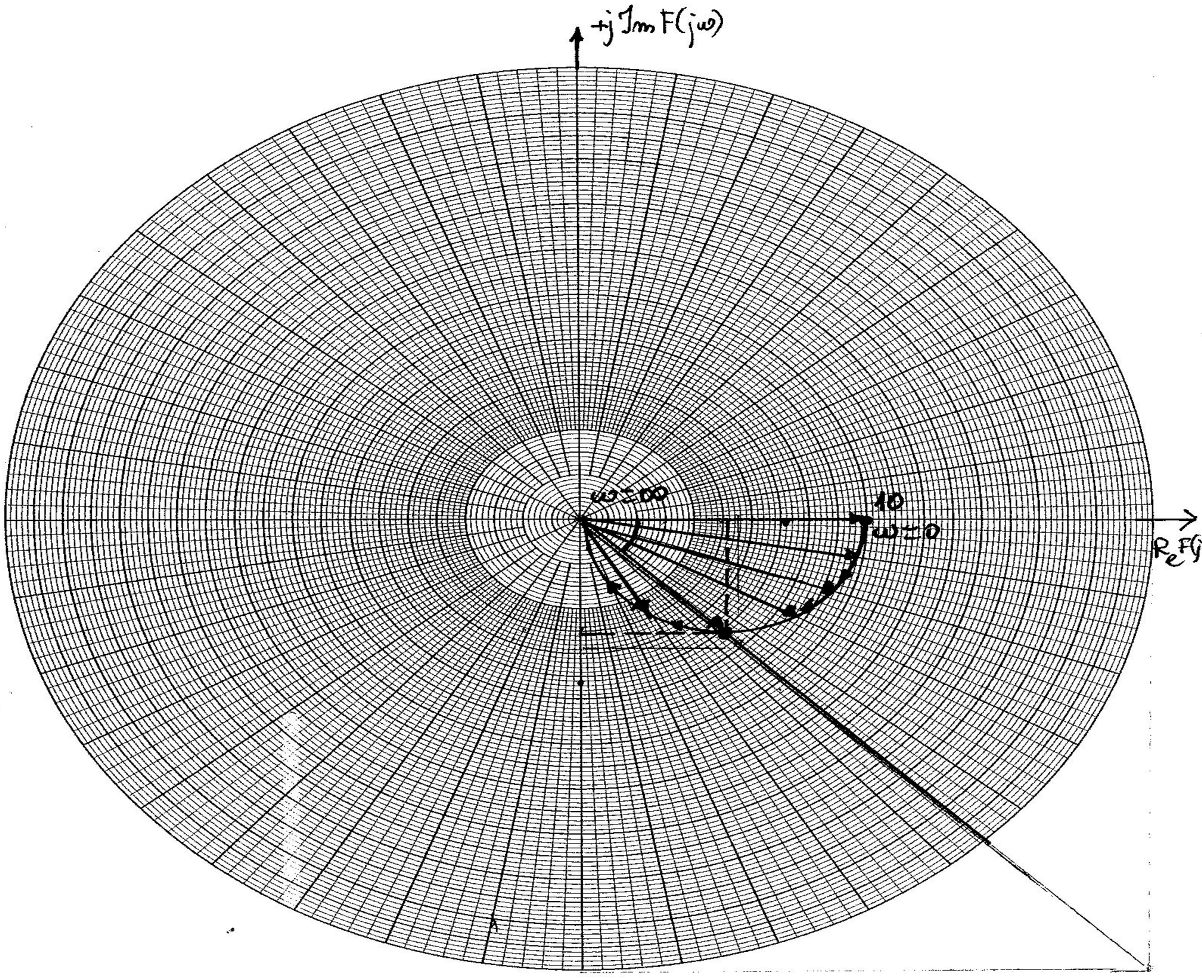
2 poli reali semplici
2 poli nulli

$$|F(j\omega)| = \frac{K}{\omega^2\sqrt{1-(\tau_1\omega)^2}\sqrt{1+(\tau_2\omega)^2}}$$

$$\varphi(\omega) = -180° - \text{arctg}\,\omega\tau_1 - \text{arctg}\,\omega\tau_2$$

ω	$\lvert F(j\omega)\rvert$	$\varphi(\omega)$
0	∞	$-180°$
∞	0	$-360°$




+j Im F(jω)
ω=∞
10
ω=0
Re F(jω)