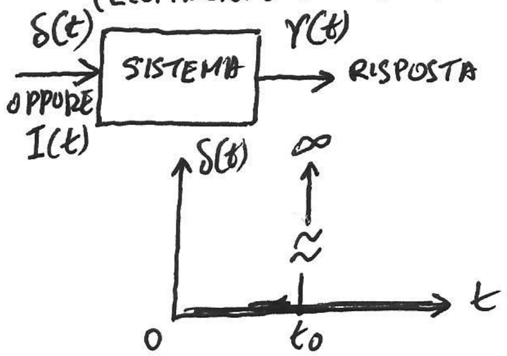
STABILITÀ DI UN SISTEMA DI CONTROLLO AUTOMATICO

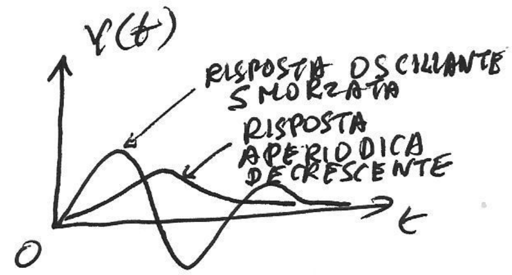
UN SISTEMA DI CONTROLLO È STABILE SE LA SUA RISPOSTA AD UN IMPULSO UNITARIO (DELTA(δ) DI DIRAC) TENDE A ZERO, OPPURE SE LA SUA RISPOSTA AL GRADINO UNITARIO u(t) RISULTA LIMITATA.

SEGNALE DI PROVA APPLICATO ALL'INGRESSO	RISPOSTA FORNITA DAL SISTEMA

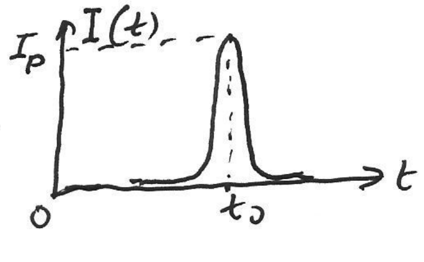
SISTEMA STABILE

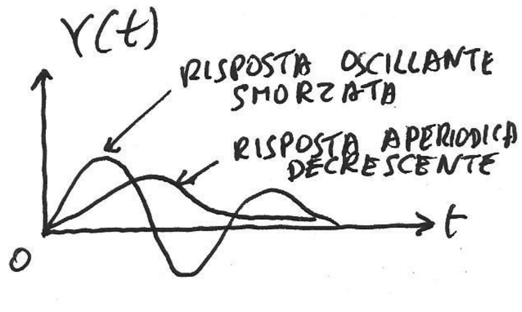
1^o CASO – (ECCITAZIONE IMPULSIVA)





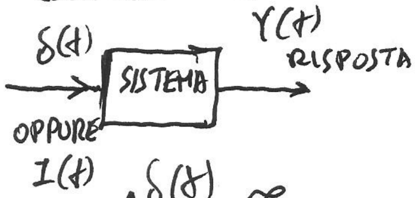
IN PRATICA, AL POSTO DEL SEGNALE IDEALE $\delta(t)$ SI PUÒ APPLICARE UN IMPULSO DI AMPIEZZA LIMITATA (I_P)

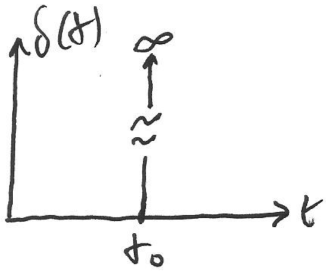




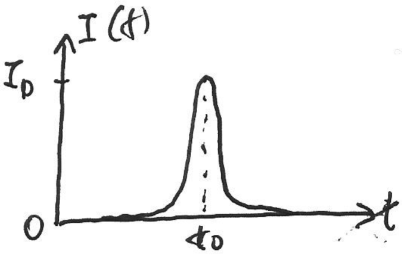
1

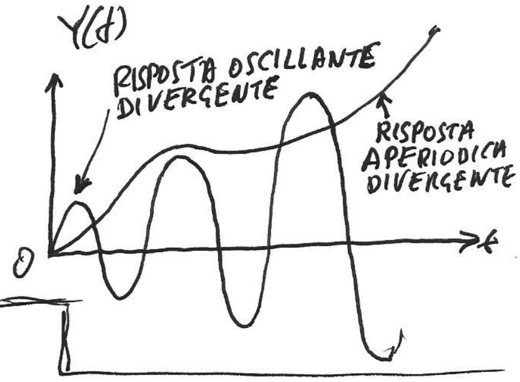
1° CASO — SISTEMA INSTABILE
(ECCITAZIONE IMPULSIVA)





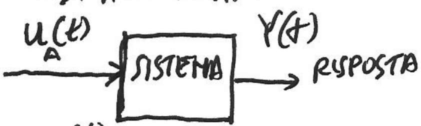
OPPURE I(t), IMPULSO DI AMPIEZZA LIMITATA (I_p)

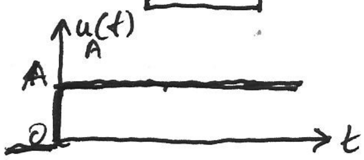


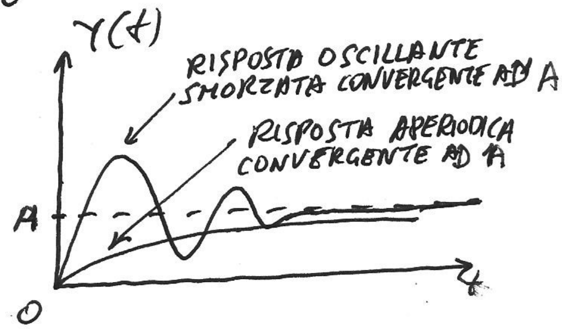


2° CASO
(ECCITAZIONE A GRADINO DI AMPIEZZA A)

SISTEMA STABILE



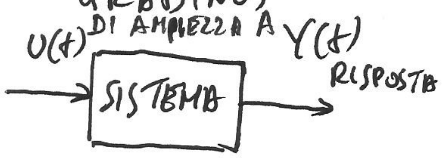


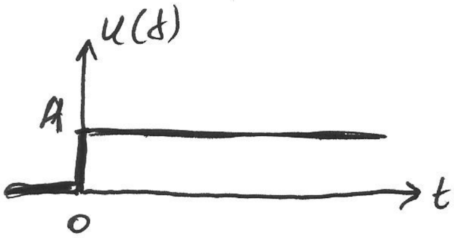


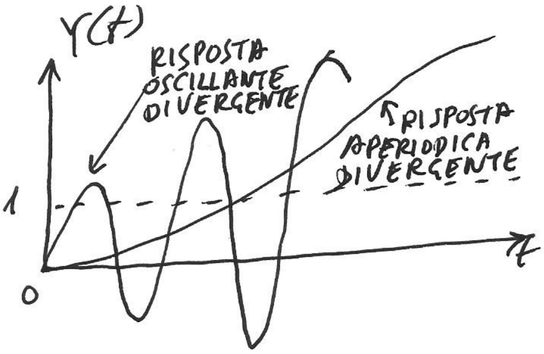
2

SISTEMA INSTABILE

2° CASO
(ECCITAZIONE A GRADINO)



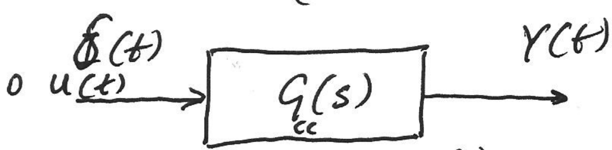




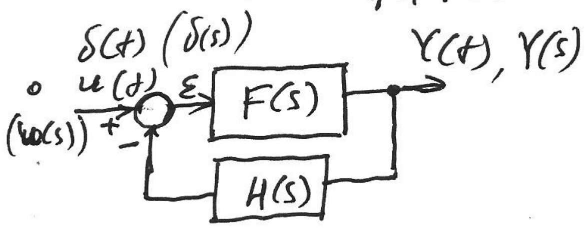
CONDIZIONE NECESSARIA PER LA STABILITÀ:

ASSENZA DI POLI REALI POSITIVI O DI POLI COMPLESSI CONIUGATI CON PARTE REALE POSITIVA $(\alpha \pm j\omega)$ CON $\alpha > 0$

NELLA FUNZIONE DI TRASFERIMENTO DEL SISTEMA (A CATENA CHIUSA) $\left(G_{cc}(s)\right)$



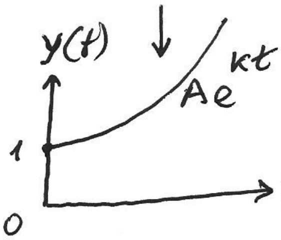
$$G_{cc}(s) = \frac{F(s)}{1 + F(s)H(s)}$$

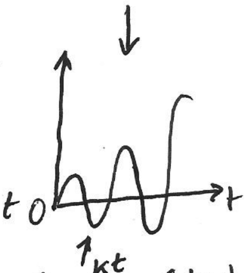


NELLA TRASFORMATA $Y(s)$ DELLA RISPOSTA $Y(t)$ NON DEVONO ESSERE PRESENTI TERMINI DEL TIPO: (CON $K>0$)

$$\frac{A}{s-K} \quad \text{o} \quad \frac{A}{s-K\pm j\omega}$$

$\downarrow$ $\downarrow$





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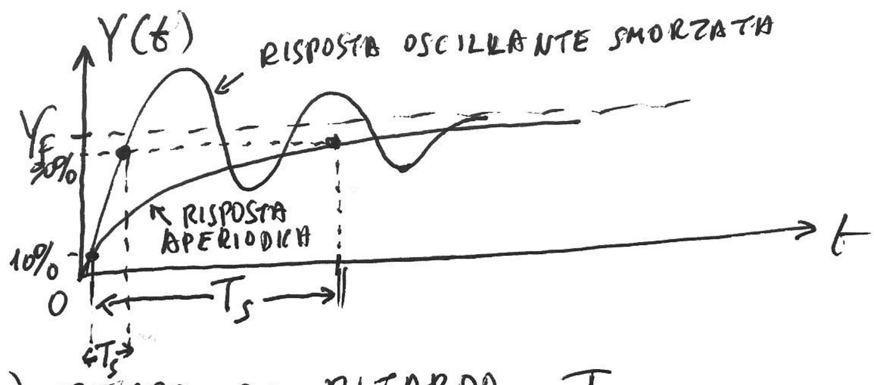
PRECISIONE DINAMICA DI
UN SISTEMA DI CONTROLLO

INDICI DI QUALITÀ:

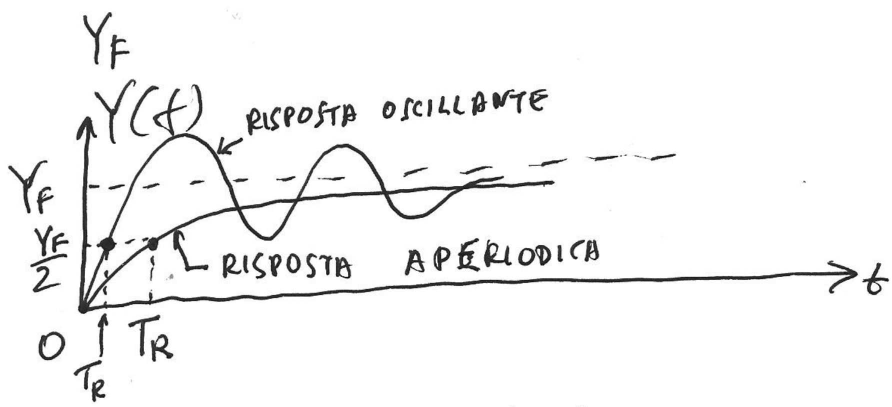
1) <u>TEMPO DI SALITA</u> T_S

TEMPO IMPIEGATO DALLA RISPOSTA $Y(t)$ PER PASSARE DAL 10% $(0{,}1\,Y_F)$ AL 90% $(0{,}9\,Y_F)$ DEL VALORE FINALE (Y_F)



2) <u>TEMPO DI RITARDO</u> T_R

TEMPO IMPIEGATO DALLA RISPOSTA $Y(t)$ PER RAGGIUNGERE IL 50% DEL VALORE FINALE

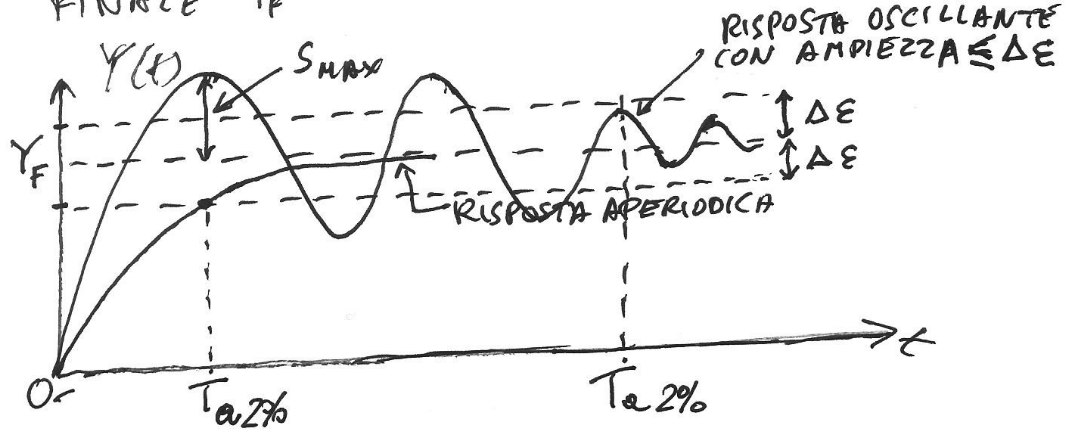


4

3) TEMPO DI ASSESTAMENTO $T_{a2\%}$

(DURATA DEL TRANSITORIO)

TEMPO IMPIEGATO DALLA RISPOSTA $Y(t)$ PER ENTRARE STABILMENTE IN UNA FASCIA DI VALORE PREFISSATO ($\pm\Delta\varepsilon = \pm 2\%$) INTORNO AL VALORE FINALE Y_F



4) SOVRAOSCILLAZIONE PERCENTUALE (OVERSHOOT) OS %

$$OS\% = \frac{S_{MAX}}{Y_F} \cdot 100$$

(VALE SOLTANTO PER RISPOSTE OSCILLANTI)

RELAZIONI VALIDE PER UN SISTEMA DEL SECONDO ORDINE $F(s) = \frac{Y(s)}{X(s)} = \frac{K_0}{s^2 + 2\zeta\omega_m s + \omega_m^2}$

POLI: $\omega_m\left(-\zeta + \sqrt{\zeta^2 - 1}\right)$, $\omega_m\left(-\zeta - \sqrt{\zeta^2 - 1}\right)$

ζ : FATTORE DI SMORZAMENTO (O COEFFICIENTE)

SE $\zeta > 1$ I POLI SONO REALI E DISTINTI

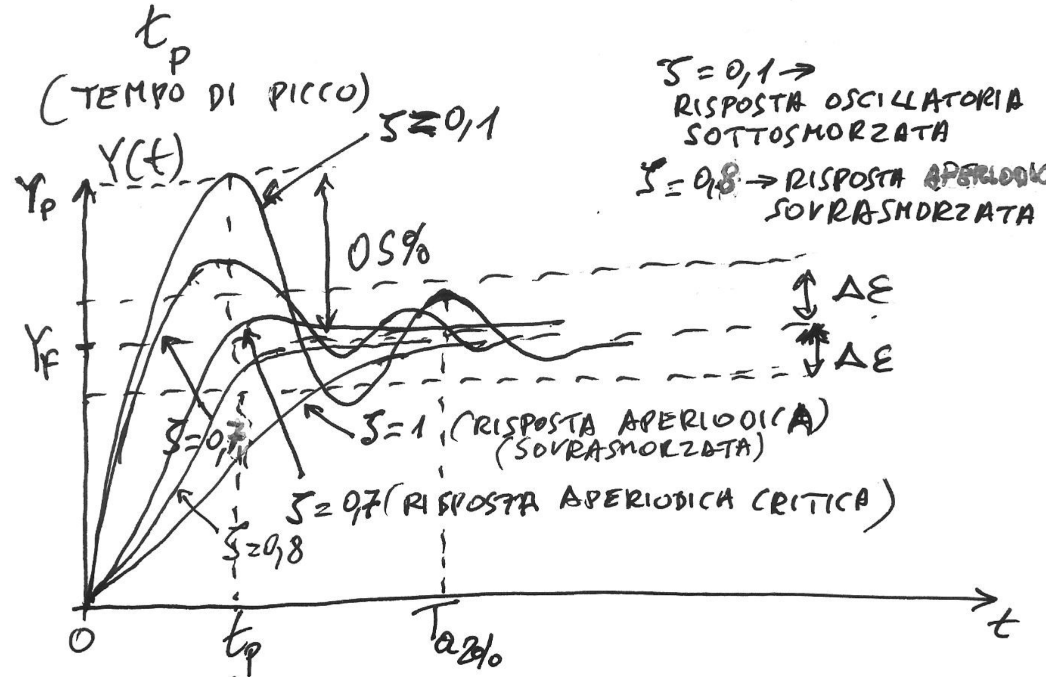
SE $\zeta = 1$ I POLI SONO REALI E COINCIDENTI

SE $\zeta < 1$ I POLI SONO COMPLESSI CONIUGATI

ω_m E' LA PULSAZIONE NATURALE NON SMORZATA

5

FORMULE APPROSSIMATE (PRATICHE)



TEMPO NECESSARIO PERCHÉ $Y(t)$ RAGGIUNGA IL VALORE DI PICCO Y_p

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$OS\% = e^{\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \cdot 100$$

$$T_{a2\%} = \frac{4}{\zeta\,\omega_n}$$

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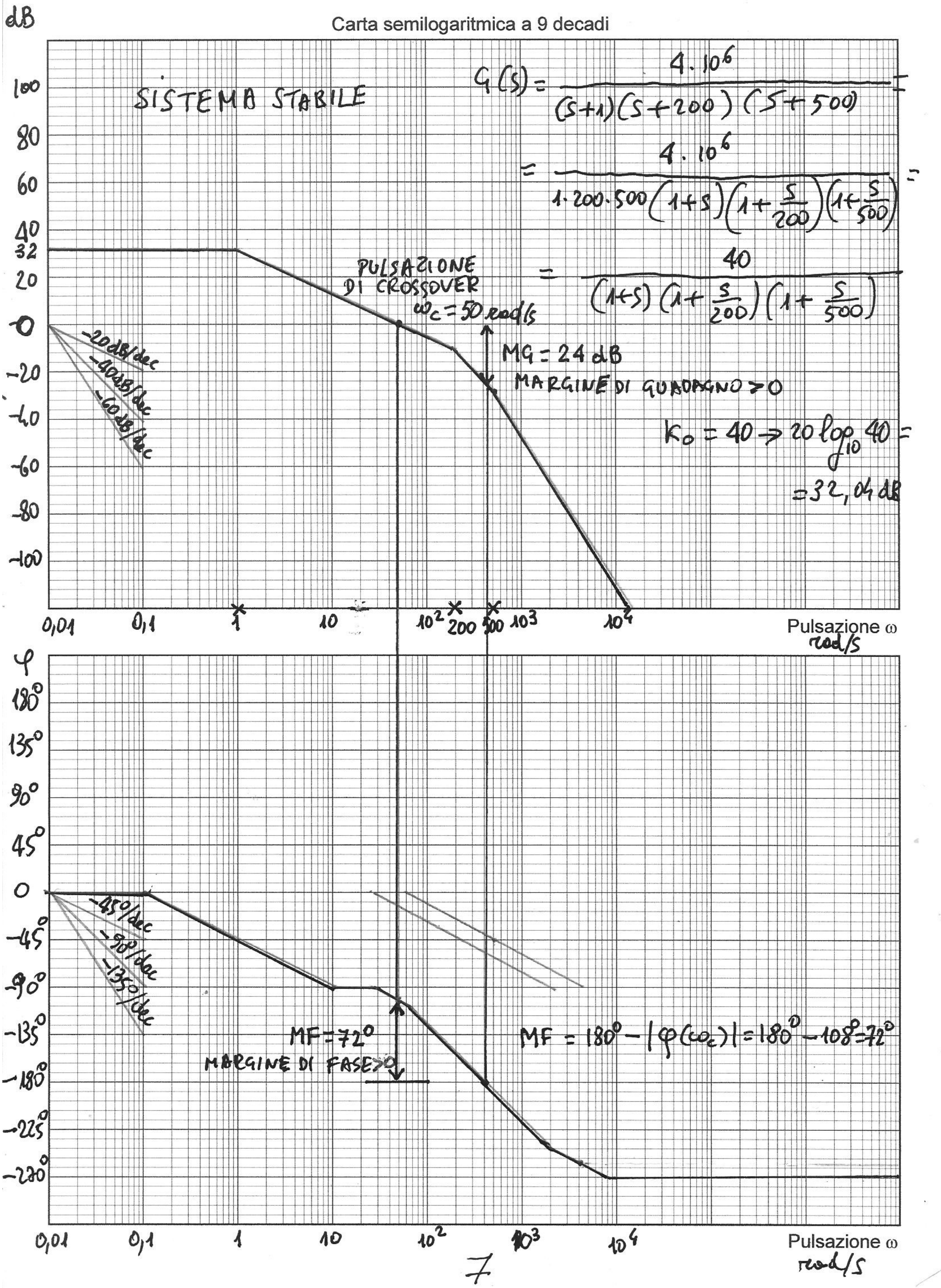
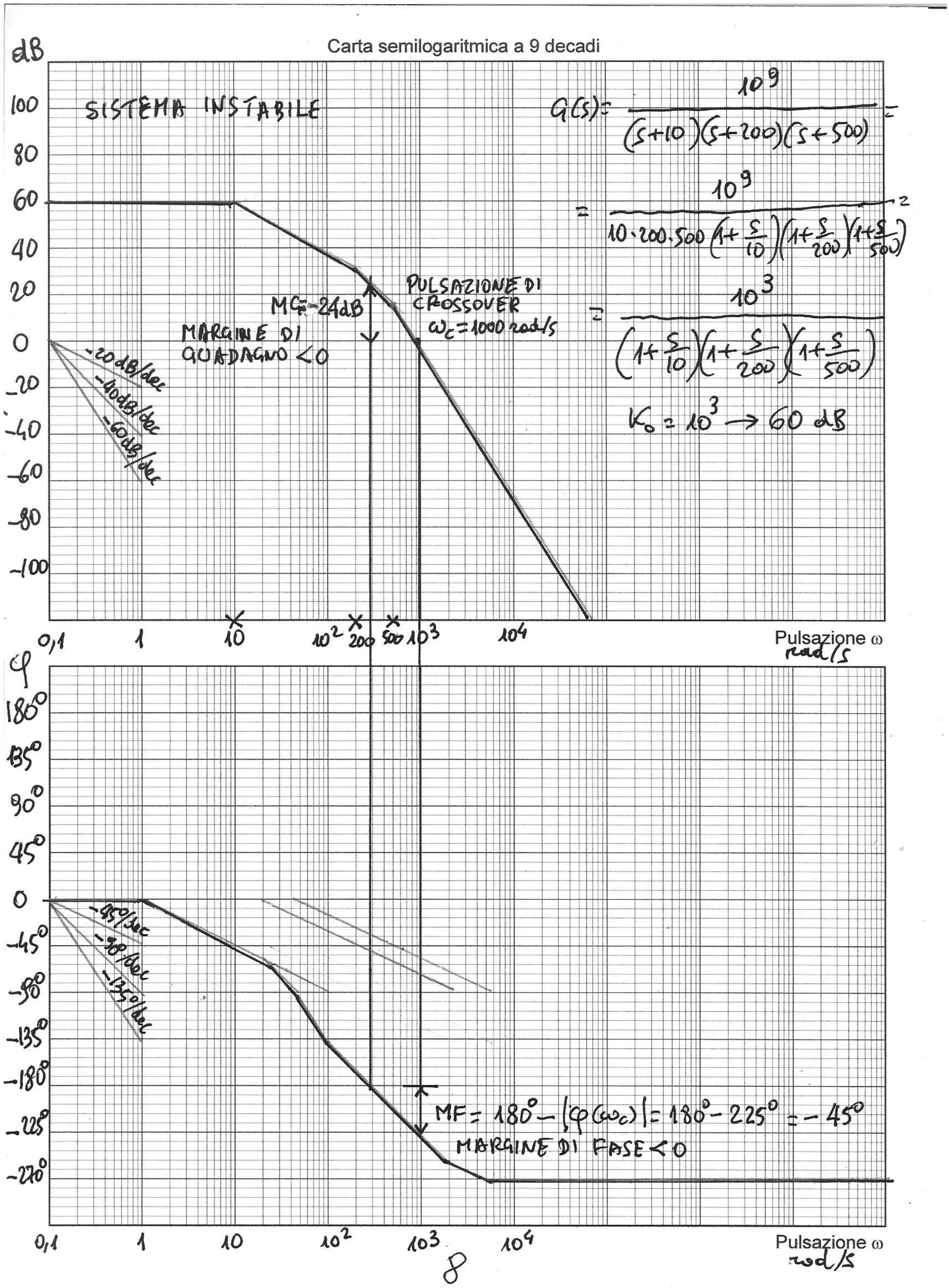
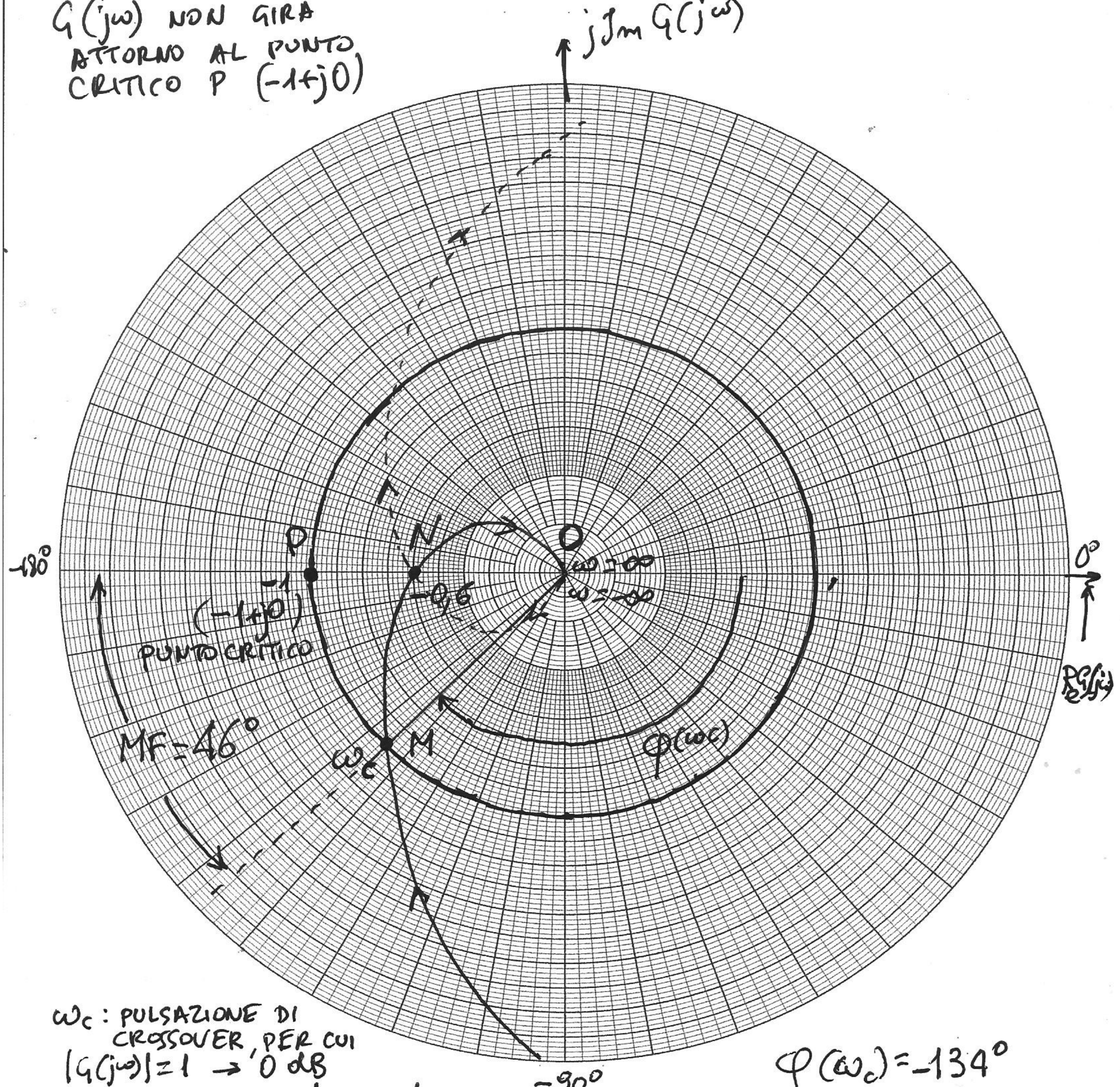

dB
Carta semilogaritmica a 9 decadi
SISTEMA STABILE
G(s) = 4·10^6 / ((s+1)(s+200)(s+500)) = 4·10^6 / (1·200·500 (1+s)(1+s/200)(1+s/500)) = 40 / ((1+s)(1+s/200)(1+s/500))
PULSAZIONE DI CROSSOVER
ωc = 50 rad/s
MG = 24 dB
MARGINE DI GUADAGNO > 0
K0 = 40 → 20 log10 40 = 32,04 dB
-20dB/dec
-40dB/dec
-60dB/dec
100
80
60
40
32
20
0
-20
-40
-60
-80
-100
0,01
0,1
1
10
10^2
200
500
10^3
10^4
Pulsazione ω
rad/s
φ
180°
135°
90°
45°
0
-45°
-90°
-135°
-180°
-225°
-270°
-45°/dec
-90°/dec
-135°/dec
MF = 72°
MARGINE DI FASE > 0
MF = 180° − |φ(ωc)| = 180° − 108° = 72°
0,01
0,1
1
10
10^2
10^3
10^4
Pulsazione ω
rad/s


Carta semilogaritmica a 9 decadi
dB
SISTEMA INSTABILE
$G(s) = \frac{10^9}{(s+10)(s+200)(s+500)} = \frac{10^9}{10\cdot200\cdot500\left(1+\frac{s}{10}\right)\left(1+\frac{s}{200}\right)\left(1+\frac{s}{500}\right)} = \frac{10^3}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{200}\right)\left(1+\frac{s}{500}\right)}$
$K_0 = 10^3 \rightarrow 60$ dB
MG = 24dB
MARGINE DI GUADAGNO <0
PULSAZIONE DI CROSSOVER
$\omega_c = 1000$ rad/s
-20dB/dec
-40dB/dec
-60dB/dec
100
80
60
40
20
0
-20
-40
-60
-80
-100
0,1
1
10
10^2
200
500
10^3
10^4
Pulsazione ω rad/s
φ
180°
135°
90°
45°
-45°
-90°
-135°
-180°
-225°
-270°
-45°/dec
-90°/dec
-135°/dec
$MF = 180° - |\varphi(\omega_c)| = 180° - 225° = -45°$
MARGINE DI FASE <0
0,1
1
10
10^2
10^3
10^4
Pulsazione ω rad/s
8

IL GRAFICO DI $G(j\omega)$ NON GIRA ATTORNO AL PUNTO CRITICO P $(-1+j0)$

SISTEMA STABILE (CON 3 POLI)



ω_c : PULSAZIONE DI CROSSOVER, PER CUI $|G(j\omega)|=1 \rightarrow 0\ dB$

$$MG = \frac{1}{\overline{ON}} = \frac{1}{0,6} \cong 1,67$$

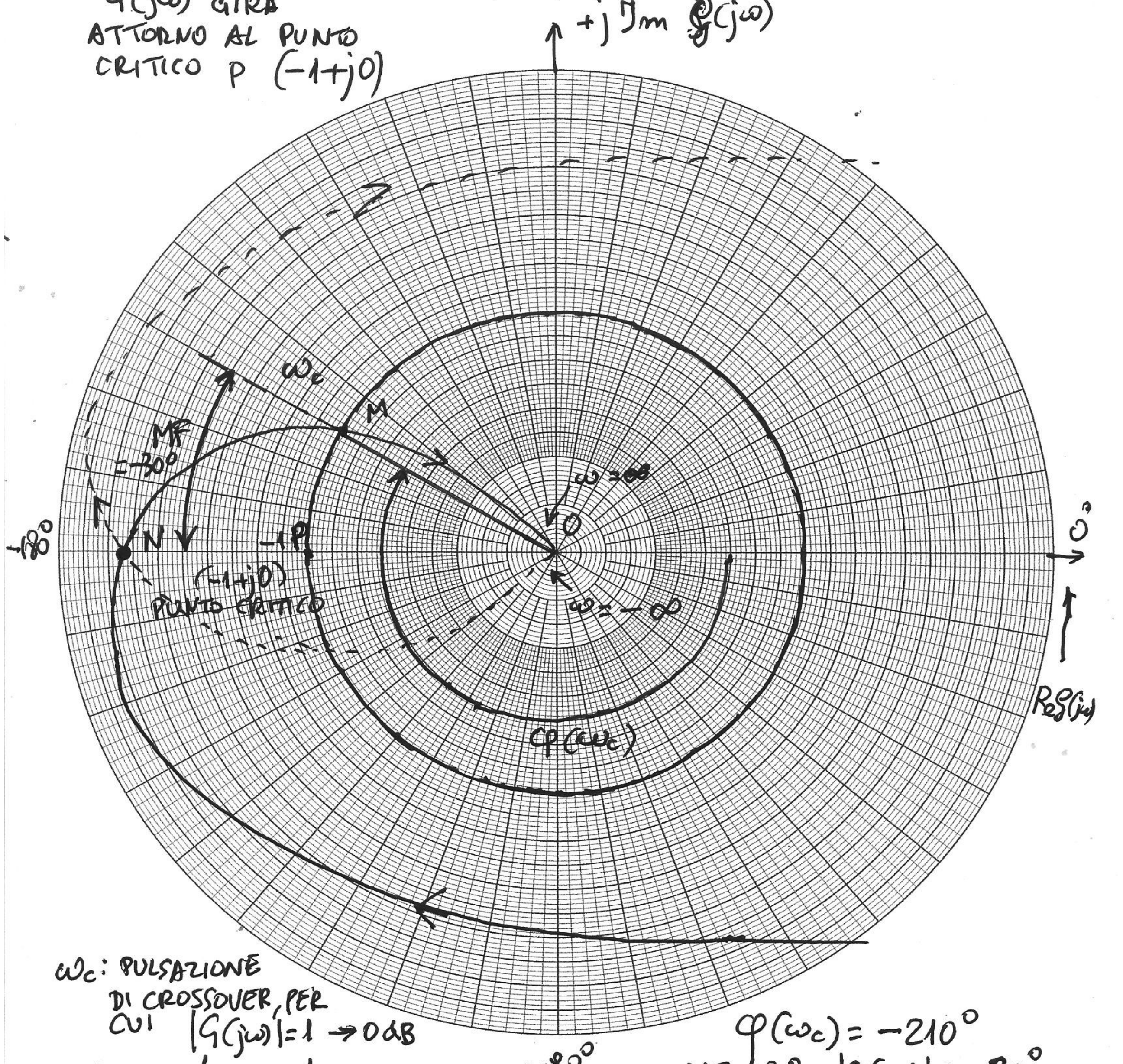
$$MG_{dB} = 20 \log_{10} MG = -20 \log_{10} \overline{ON} = -20 \log_{10} 0,6 = 4,43\ dB \ ; \ MG_{dB} > 0\ dB$$

$$\varphi(\omega_c) = -134^\circ$$

$$MF = 180^\circ - |\varphi(\omega_c)| = 46^\circ \ ; \ MF > 0^\circ$$

SISTEMA INSTABILE (CON 3 POLI)

IL GRAFICO DI $G(j\omega)$ GIRA ATTORNO AL PUNTO CRITICO P $(-1+j0)$



ω_c: PULSAZIONE DI CROSSOVER, PER CUI $|G(j\omega)| = 1 \rightarrow 0\,dB$

$$MG = \frac{1}{\overline{ON}} = \frac{1}{1{,}74} \simeq 0{,}574$$

$$\varphi(\omega_c) = -210^\circ$$

$$MF = 180^\circ - |\varphi(\omega_c)| = -30^\circ; \quad MF < 0^\circ$$

$$MG_{dB} = 20 \log_{10} MG = -20 \log_{10} \overline{ON} = -20 \log_{10} 1{,}74 = -4{,}81\,dB$$

$$MG < 0\,dB$$

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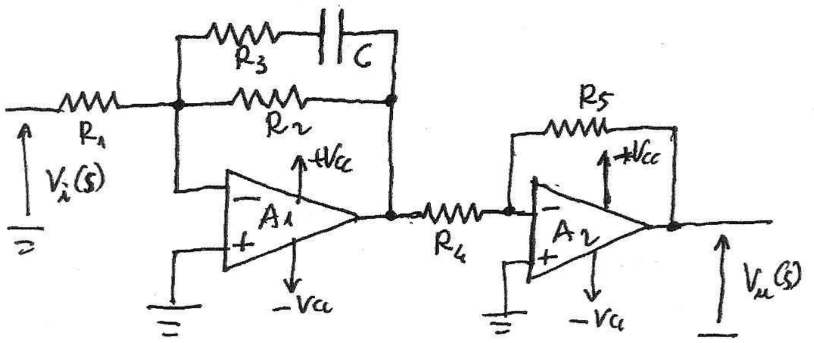
RETI CORRETTRICI

RETE RITARDATRICE ATTIVA NON INVERTENTE 1



$$Z_R(s) = R_2 \,//\, \left(R_3 + \frac{1}{sC}\right) = \frac{R_2 \cdot \left(R_3 + \frac{1}{sC}\right)}{R_2 + R_3 + \frac{1}{sC}} = \frac{\frac{R_2\,(R_3Cs+1)}{\cancel{sC}}}{\frac{(R_2+R_3)Cs+1}{\cancel{sC}}} =$$

$$= \frac{R_2\,(R_3Cs+1)}{(R_2+R_3)Cs+1}$$

$$G(s) = \frac{V_u(s)}{V_i(s)} = \left(-\frac{R_5}{R_4}\right)\left(-\frac{Z_R(s)}{R_1}\right) = \frac{R_5}{R_4R_1}\,\frac{R_2\,(R_3Cs+1)}{[(R_2+R_3)Cs+1]}$$

Se $R_4 = R_5$ e $R_1 = R_2$ si ha:

$$G(s) = \frac{R_2\,(R_3Cs+1)}{R_1[(R_2+R_3)Cs+1]} = \frac{R_3Cs+1}{(R_2+R_3)Cs+1} =$$

$$= \frac{R_3Cs+1}{(R_2+R_3)Cs+1}$$

$$\tau_z = R_3C \qquad \omega_z = \frac{1}{\tau_z} = \frac{1}{R_3C}$$

$$\tau_p = (R_2+R_3)C \qquad \omega_p = \frac{1}{\tau_p} = \frac{1}{(R_2+R_3)C}$$

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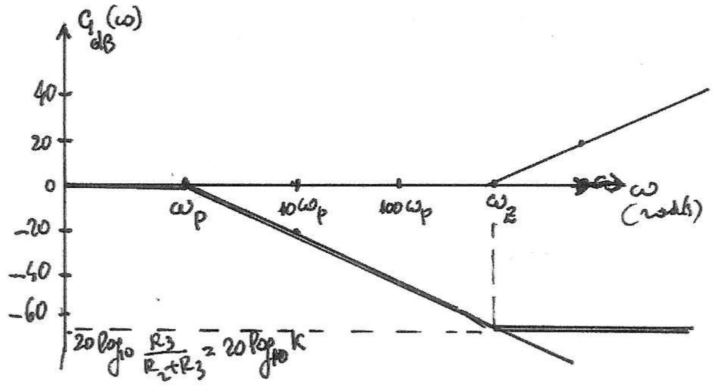
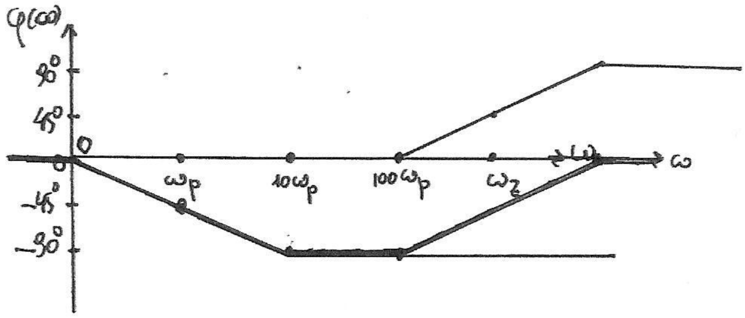
2



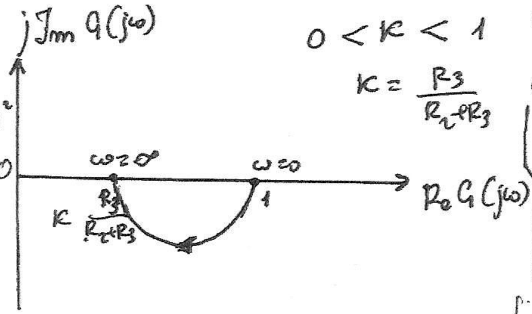


$0 < K < 1$

$K = \frac{R_3}{R_2+R_3}$

$\omega_p = \frac{1}{(R_2+R_3)C}$

$|G(j\omega)| = \frac{\sqrt{1+(\omega R_3 C)^2}}{\sqrt{1+[\omega(R_2+R_3)C]^2}}$



$$\omega_z = \frac{1}{R_3 C} = \frac{(R_2+R_3)}{(R_2+R_3)C R_3} = \frac{\omega_p}{K}$$

$$\boxed{\omega_p = K\omega_z}$$

ω	$\lvert G(j\omega)\rvert$	$\varphi(\omega)$
0	1	0
∞	$\frac{R_3}{R_2+R_3}$	0

$\varphi(\omega) = \mathrm{arctg}\ \omega R_3 C - \mathrm{arctg}\ \omega(R_2+R_3)C$

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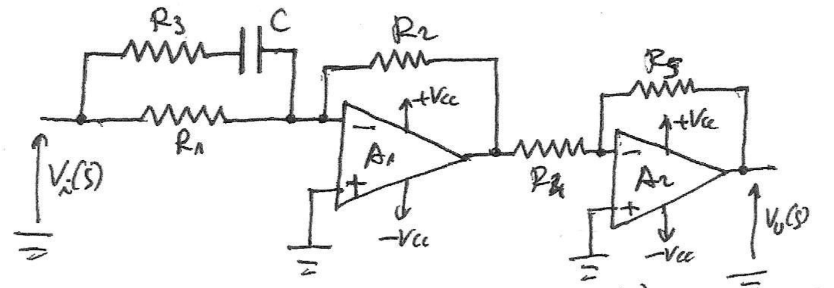
RETE ANTICIPATRICE ATTIVA NON INVERTENTE 3



$$Z_i(s) = R_1 // \left(R_3 + \frac{1}{sC}\right) = \frac{R_1\left(R_3 + \frac{1}{sC}\right)}{R_1 + R_3 + \frac{1}{sC}} = \frac{R_1(R_3Cs+1)}{(R_1+R_3)Cs+1}$$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \left(-\frac{R_2}{Z_R(s)}\right)\cdot\left(-\frac{R_5}{R_4}\right) = \frac{R_5}{R_4}\cdot\frac{R_2\left[(R_2+R_3)Cs+1\right]}{R_1(R_3Cs+1)}$$

Se $R_1 = R_2$ e $R_4 = R_5$ si ha:

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1 + (R_1+R_3)Cs}{1 + R_3Cs}$$

$$\tau_z = (R_1+R_3)C \; ; \quad \tau_p = R_3C$$

$$\omega_z = \frac{1}{\tau_z} = \frac{1}{(R_1+R_3)C} \; ; \quad \omega_p = \frac{1}{\tau_p} = \frac{1}{R_3C}$$

$$K = \frac{R_1+R_3}{R_3}$$

$$K > 1$$

$$\frac{\omega_p}{\omega_z} = \frac{(R_1+R_3)C}{R_3C} = \frac{R_2+R_3}{R_3} = K$$

$$\omega_p = K\,\omega_z$$

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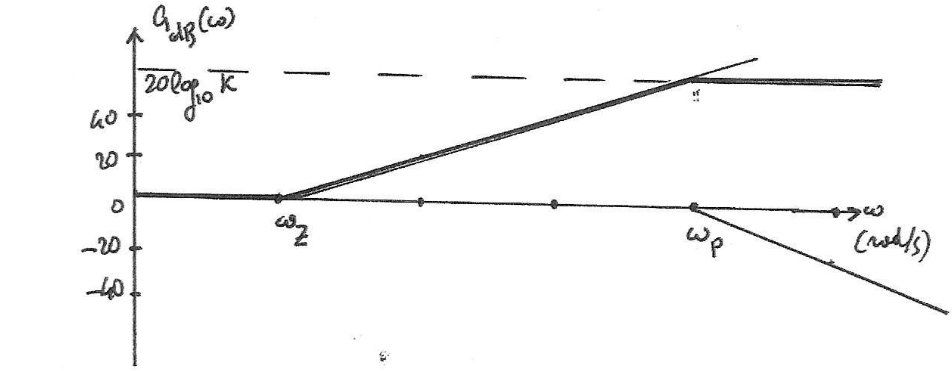
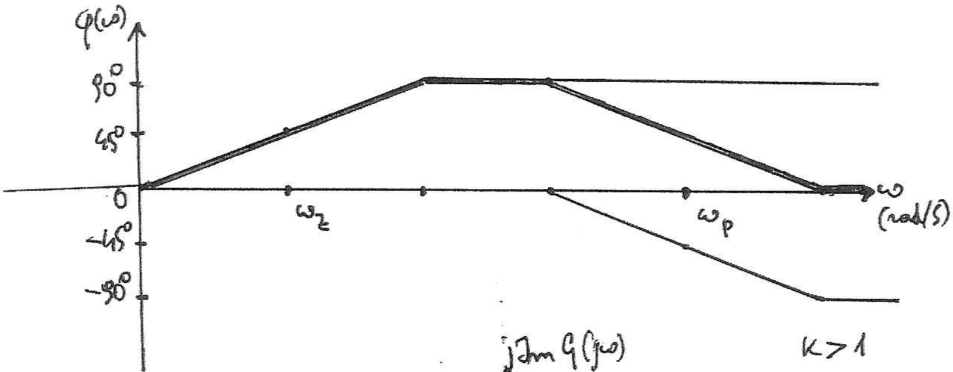
4



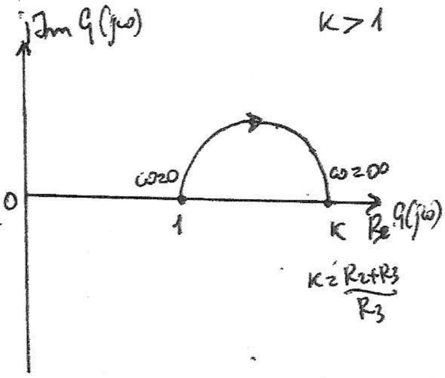


K > 1

$$|G(j\omega)| = \frac{\sqrt{1+[\omega(R_3+R_2)C]^2}}{\sqrt{1+(\omega R_3 C)^2}}$$

$$\varphi(\omega) = \operatorname{arctg}[\omega(R_2+R_3)C] - \operatorname{arctg} \omega R_3 C$$

ω	$\lvert G(j\omega)\rvert$	$\varphi(\omega)$
0	1	0
∞	$\frac{R_2+R_3}{R_3}$	0



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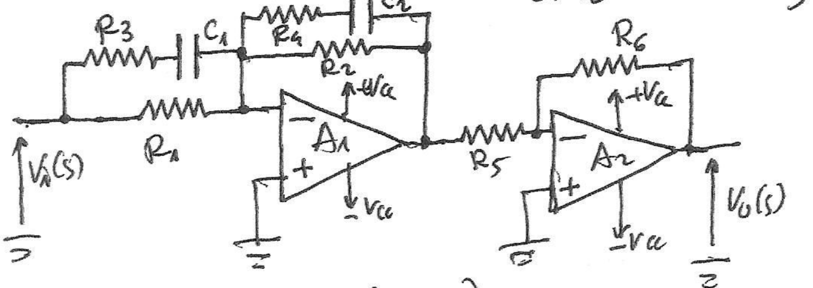
RETE A SELLA NON INVERTENTE 5



$$G(s) = \frac{V_u(s)}{V_i(s)} = \left(-\frac{Z_R(s)}{Z_i(s)}\right)\left(-\frac{R_6}{R_5}\right) \quad \text{con } R_5 = R_6$$

$$Z_R(s) = R_2 // \left(R_4 + \frac{1}{sC_2}\right) = \frac{R_2\left(R_4 + \frac{1}{sC_2}\right)}{R_2 + R_4 + \frac{1}{sC_2}} =$$

$$= \frac{R_2\,(R_4C_2s+1)}{(R_2+R_4)C_2s+1}$$

$$Z_i(s) = R_1 // \left(R_3 + \frac{1}{sC_1}\right) = \frac{R_1\left(R_3 + \frac{1}{sC_1}\right)}{R_1 + R_3 + \frac{1}{sC_1}} =$$

$$= \frac{R_1\,(R_3C_1s+1)}{(R_1+R_3)C_1s+1}$$

$$G(s) = \frac{R_2}{R_1}\cdot\frac{(R_4C_2s+1)\left[(R_1+R_3)C_1s+1\right]}{\left[(R_2+R_4)C_2s+1\right](R_3C_1s+1)}$$

$\tau_{z1} = R_4C_2$; $\omega_{z1} = \frac{1}{R_4C_2}$ $\qquad \omega_{p1}\omega_{p2} = \omega_{z1}\omega_{z2}$

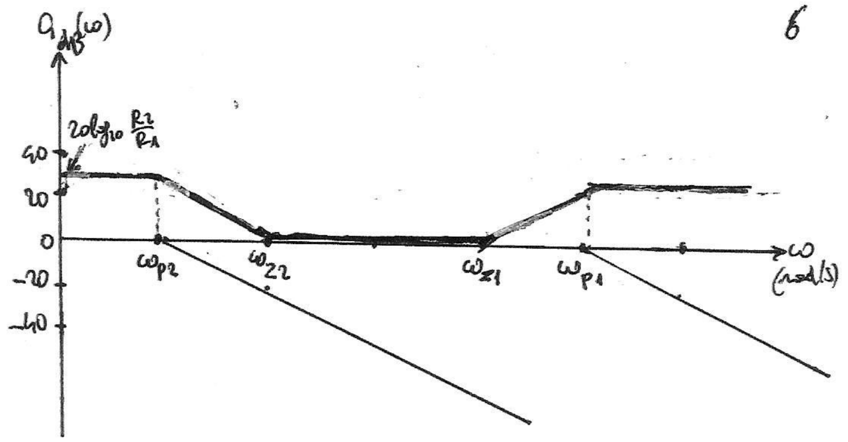
$\tau_{z2} = (R_1+R_3)C_1$; $\omega_{z2} = \frac{1}{(R_1+R_3)C_1}$ $\qquad \frac{\omega_{p1}}{\omega_{z1}} = \frac{\omega_{z2}}{\omega_{p2}}$

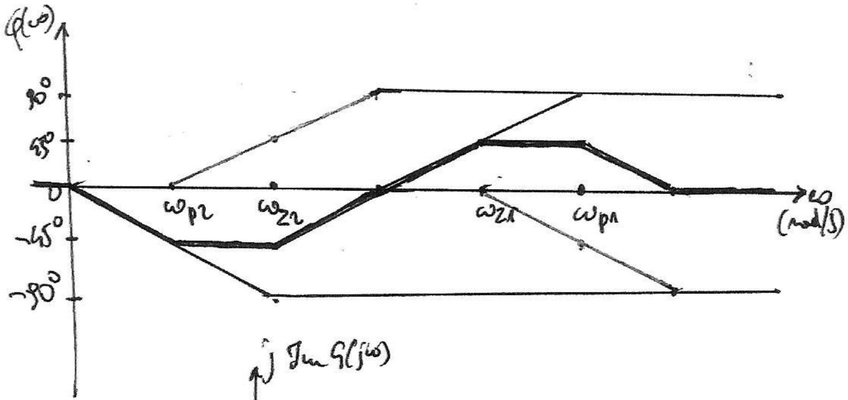
$\tau_{p1} = R_3C_1$; $\omega_{p1} = \frac{1}{R_3C_1}$

$\tau_{p2} = (R_2+R_4)C_2$; $\omega_{p2} = \frac{1}{(R_2+R_4)C_2}$

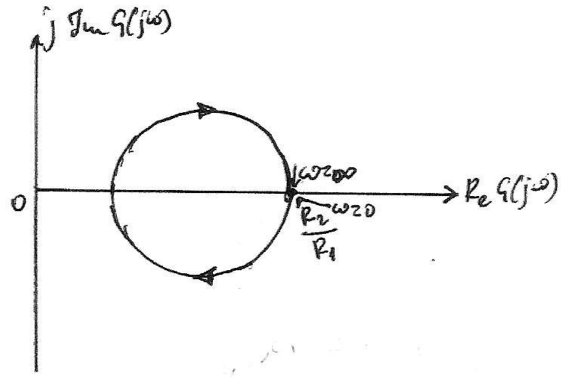
15

6





ω	$\lvert G(j\omega)\rvert$	$\varphi(\omega)$
0	R_2/R_1	0
∞	R_2/R_1	0



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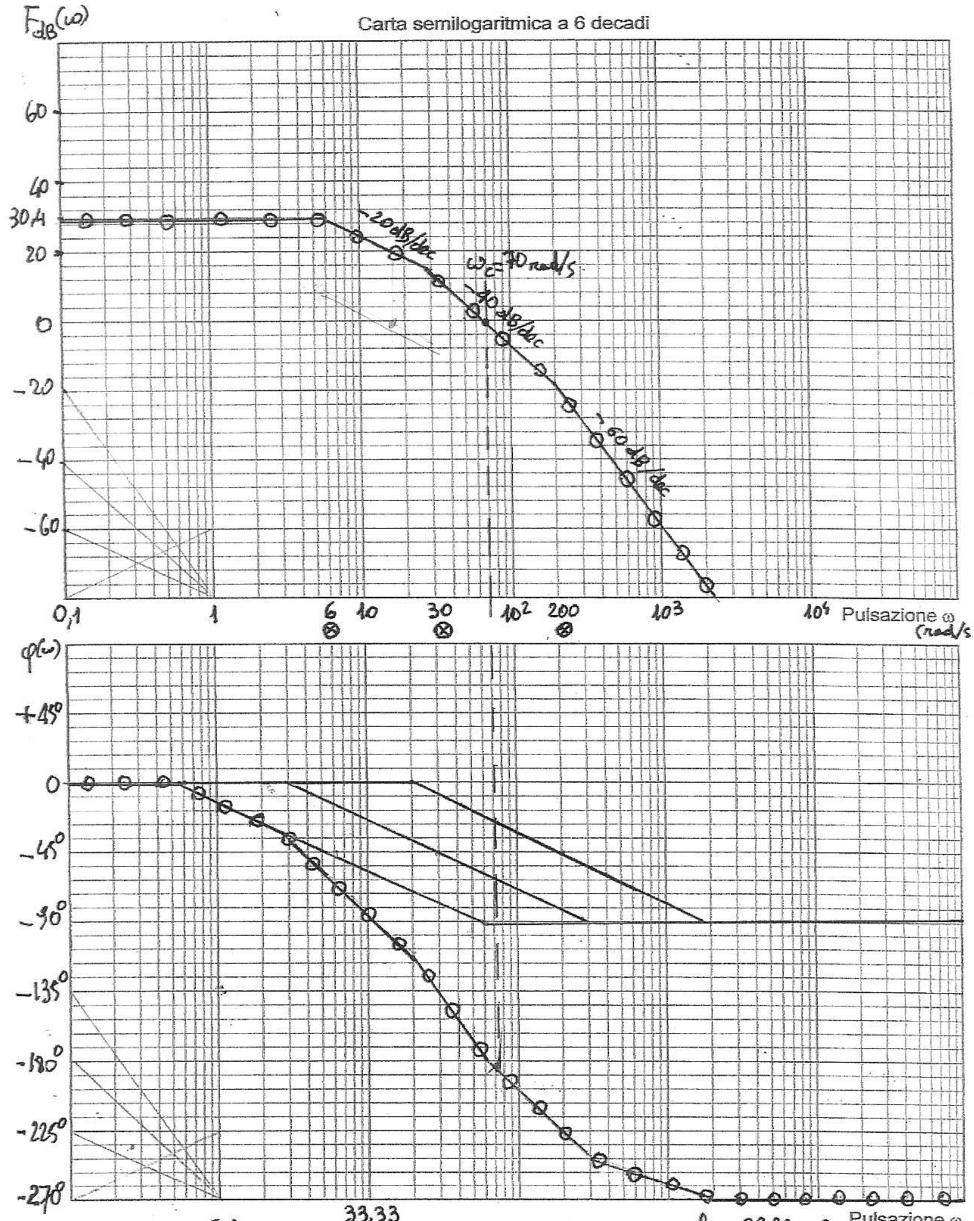
SISTEMA DA STABILIZZARE

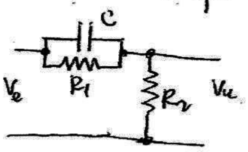


$$G_a(S) = \frac{33,33}{\left(1+\frac{S}{6}\right)\left(1+\frac{S}{30}\right)\left(1+\frac{S}{200}\right)} \qquad K_{o(dB)} = 20 \log_{10} 33,33 = 30,4 \, dB$$

17

Rete anticipatrice PASSIVA

3



$$F(s) = \frac{V_u(s)}{V_e(s)} = \frac{R_2}{R_2 + R_1 /\!/ \frac{1}{sC}} = \frac{R_2}{R_2 + \frac{R_1/sC}{R_1 + \frac{1}{sC}}} =$$

$$= \frac{R_2}{R_2 + \frac{R_1/sC}{\frac{R_1Cs+1}{sC}}} = \frac{R_2}{R_2 + \frac{R_1}{R_1Cs+1}} = \frac{R_1R_2Cs + R_2}{R_1R_2Cs + R_2 + R_1} =$$

$$= \frac{\cancel{R_2}(R_1Cs+1)}{\cancel{R_2}\left(R_1Cs + \frac{R_2+R_1}{R_2}\right)} = \frac{\cancel{R_1C}\left(s + \frac{1}{R_1C}\right)}{\cancel{R_1C}\left(s + \frac{R_2+R_1}{R_1R_2C}\right)} = \frac{\left(Ks + \frac{1}{\tau}\right)}{\left(s + \frac{1}{K\tau}\right)} \quad \Bigg| \quad K = \frac{R_2}{R_1+R_2} \quad \tau = R_1C$$

$$F(j\omega) = \frac{K(1+j\omega\tau)}{(1+j\omega K\tau)}, \qquad \lim_{\omega \to \infty} F(j\omega) = K\frac{\tau}{K\tau} = 1$$

$$\varphi(\omega) = \text{arctg}\,\omega\tau - \text{arctg}\,\omega K\tau \qquad \lim_{\omega \to 0} F(j\omega) = K \qquad 0 < K < 1$$

$$F_{dB}(j\omega) = 20\log_{10}K + 20\log_{10}\sqrt{1+\omega^2\tau^2} - 20\log_{10}\sqrt{1+\omega^2K^2\tau^2}$$

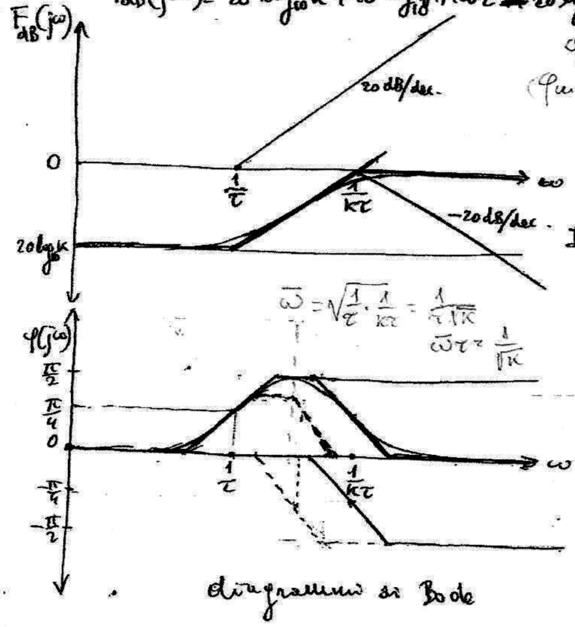


diagramma di Bode

$$\varphi(\bar{\omega}) = \text{arctg}\,\bar{\omega}\tau - \text{arctg}\,\bar{\omega}K\tau$$

$$(\varphi_{max}) = \text{arctg}\,\frac{1}{\sqrt{K}} - \text{arctg}\,\frac{K}{\sqrt{K}} =$$

$$= \text{arctg}\,\frac{1}{\sqrt{K}} - \text{arctg}\,\sqrt{K}$$

Se $K = 0$

$$\varphi(\bar{\omega}) = \text{arctg}\,\infty - \text{arctg}\,0 = 90°$$

Se $K = 1$

$$\varphi(\bar{\omega}) = \text{arctg}\,1 - \text{arctg}\,1 = 0°$$

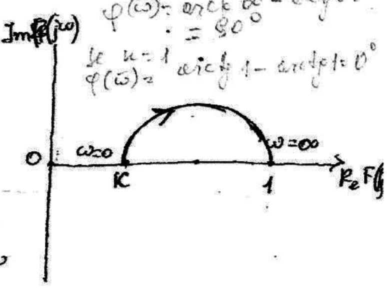
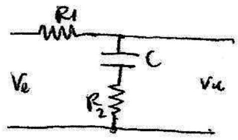


diagramma di Nyquist

18

Rete ritardatrice PASSIVA 4



$$F(s) = \frac{V_u(s)}{V_e(s)} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}} = \frac{R_2 C s + 1}{(R_1+R_2)Cs + 1} =$$

$$= \frac{R_2 C\left(s + \frac{1}{R_2 C}\right)}{(R_1+R_2)C\left[s + \frac{1}{(R_1+R_2)C}\right]} = \frac{1}{K}\frac{\left(s+\frac{1}{\tau}\right)}{\left(s+\frac{1}{K\tau}\right)} =$$

$R_2 C = \tau$

$\frac{R_2}{R_1+R_2} = \frac{1}{K}$

$K = \frac{R_1+R_2}{R_2} > 1$

$$F(j\omega) = \frac{1}{K}\frac{\frac{(1+j\omega\tau)}{\tau}}{\frac{(1+j\omega K\tau)}{K\tau}} = \frac{1+j\omega\tau}{1+j\omega K\tau}$$

$$\frac{1}{(R_1+R_2)C} = \frac{1}{KR_2C} = \frac{1}{K\tau}$$

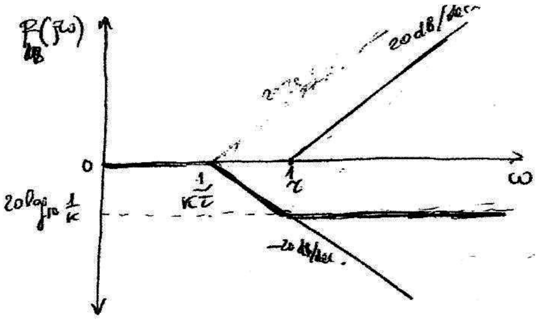
$$\lim_{\omega\to\infty} F(j\omega) = \frac{1}{K}$$

$\varphi(\omega) = \operatorname{arctg}\omega\tau - \operatorname{arctg}\omega K\tau$

$F_{dB}(\omega) = 20\log_{10}\sqrt{1+\omega^2\tau^2} - 20\log_{10}\sqrt{1+\omega^2K^2\tau^2}$

$$\lim_{\omega\to 0} F(j\omega) = 1$$

$K>1$



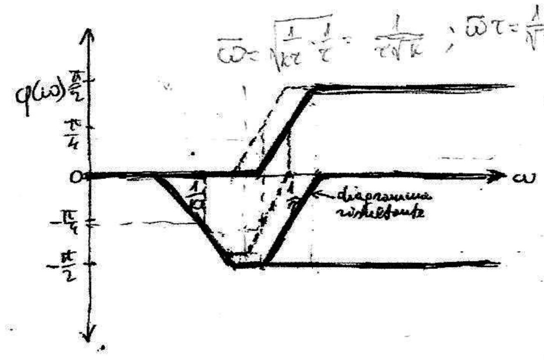
$\varphi(\bar\omega) = \operatorname{arctg}\bar\omega\tau - \operatorname{arctg}\bar\omega K\tau =$

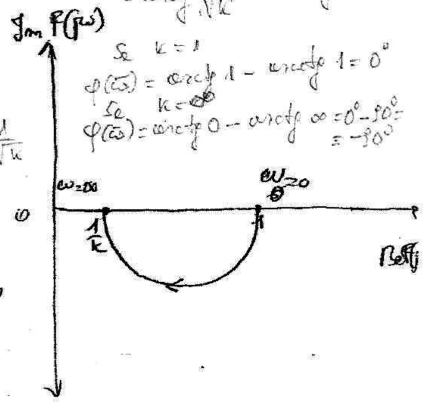
$(\varphi_{min}) = \operatorname{arctg}\frac{1}{\sqrt{K}} - \operatorname{arctg}\frac{K}{\sqrt{K}} =$

$= \operatorname{arctg}\frac{1}{\sqrt{K}} - \operatorname{arctg}\sqrt{K}$

Se $K=1$
$\varphi(\bar\omega) = \operatorname{arctg}1 - \operatorname{arctg}1 = 0°$
Se $K=\infty$
$\varphi(\bar\omega) = \operatorname{arctg}0 - \operatorname{arctg}\infty = 0° - 90° = -90°$

$\bar\omega = \sqrt{\frac{1}{K\tau}\cdot\frac{1}{\tau}} = \frac{1}{\tau\sqrt{K}}$; $\bar\omega\tau = \frac{1}{\sqrt{K}}$





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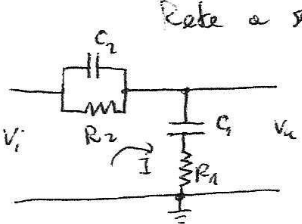
Rete a sella PASSIVA 5



$$I = \frac{V_i}{\dfrac{1}{\frac{1}{R_2}+sC_2}+R_1+\dfrac{1}{sC_1}}$$

$$V_u = I\left(R_1+\frac{1}{sC_1}\right) = \frac{V_i}{\dfrac{R_2}{1+R_2C_2s}+R_1+\dfrac{1}{sC_1}}\cdot\left(R_1+\frac{1}{sC_1}\right) =$$

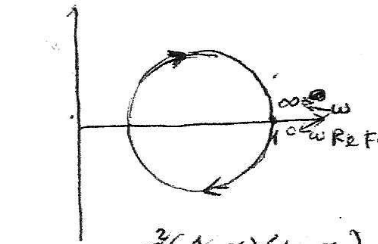
$$= \frac{V_i}{\dfrac{R_2C_1s+R_1C_1s(1+R_2C_2s)+1+R_2C_2s}{(1+R_2C_2s)sC_1}}\cdot\frac{1+R_1C_1s}{sC_1} =$$

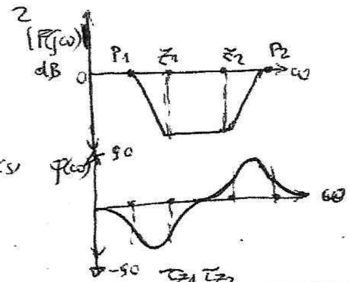
$$= \frac{V_i\,(1+R_1C_1s)(1+R_2C_2s)}{R_2C_1s+R_1C_1s+R_1C_1R_2C_2s^2+1+R_2C_2s}$$

$$F(s)=\frac{V_u}{V_i}=\frac{(1+R_1C_1s)(1+R_2C_2s)}{1+(\underbrace{R_1C_1}_{\tau_{z1}}+\underbrace{R_2C_2}_{\tau_{z2}}+\underbrace{R_2C_1}_{\tau_{21}})s+R_1C_1R_2C_2s^2}=\frac{(1+\tau_{z1}s)(1+\tau_{z2}s)}{1+(\tau_{z1}+\tau_{z2}+\tau_{21})s+\tau_{z1}\tau_{z2}s^2}$$

$$1+(\tau_{z1}+\tau_{z2}+\tau_{21})s+\tau_1\tau_2s^2=0$$

$$s=-\frac{(\tau_{z1}+\tau_{z2}+\tau_{21})\pm\sqrt{(\tau_{z1}+\tau_{z2}+\tau_{21})^2-4\tau_{z1}\tau_{z2}}}{2}$$





$$\lim_{s\to\infty}F(s)=\frac{s^2\left(\frac{1}{s}+\tau_{z1}\right)\left(\frac{1}{s}+\tau_{z2}\right)}{s^2\left[\frac{1}{s^2}+(\tau_{z1}+\tau_{z2}+\tau_{21})\frac{1}{s}+\tau_{z1}\tau_{z2}\right]}=\frac{\tau_{z1}\tau_{z2}}{\tau_{z1}\tau_{z2}}=1$$

$$\lim_{s\to 0}F(s)=1$$

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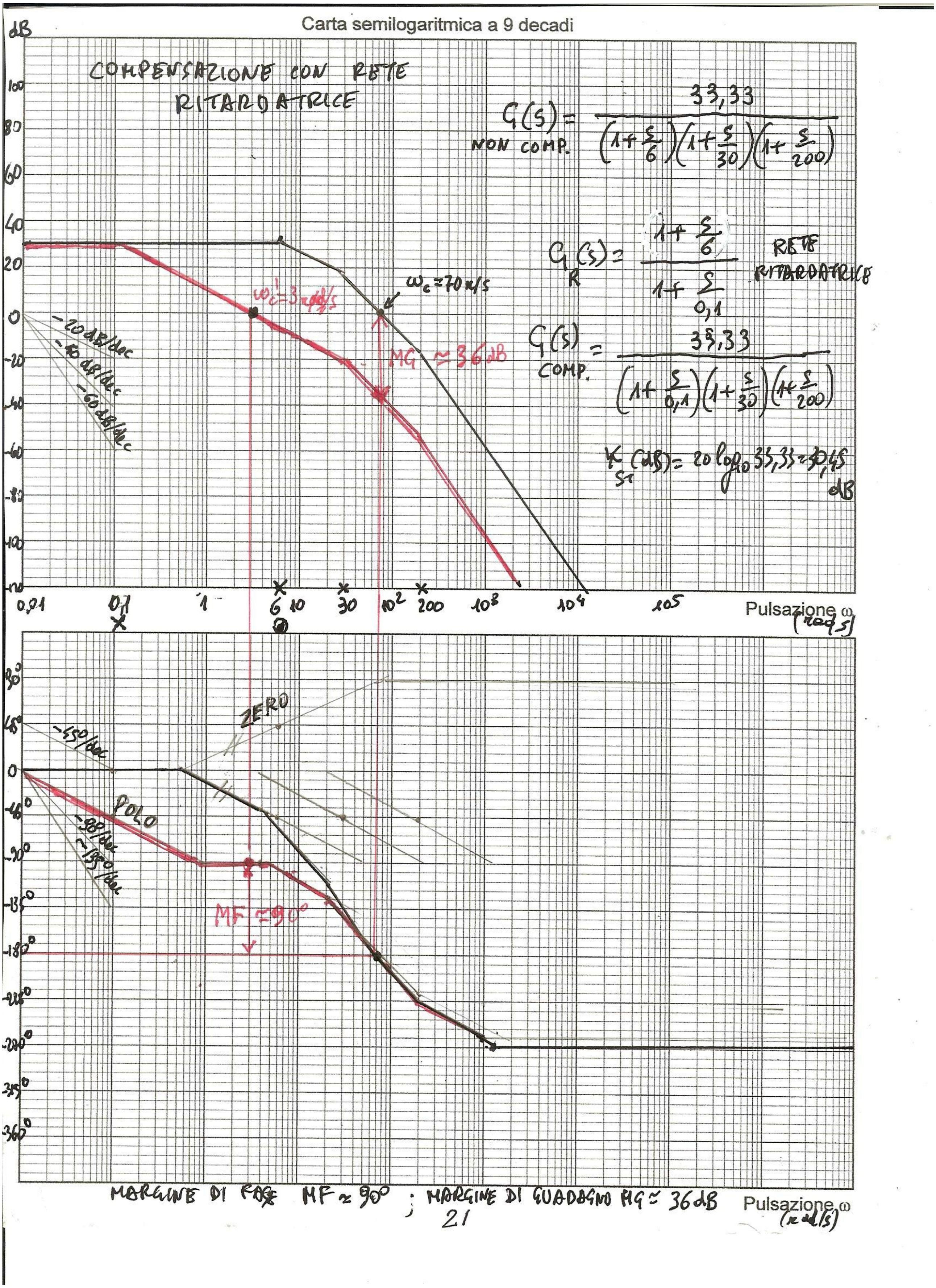
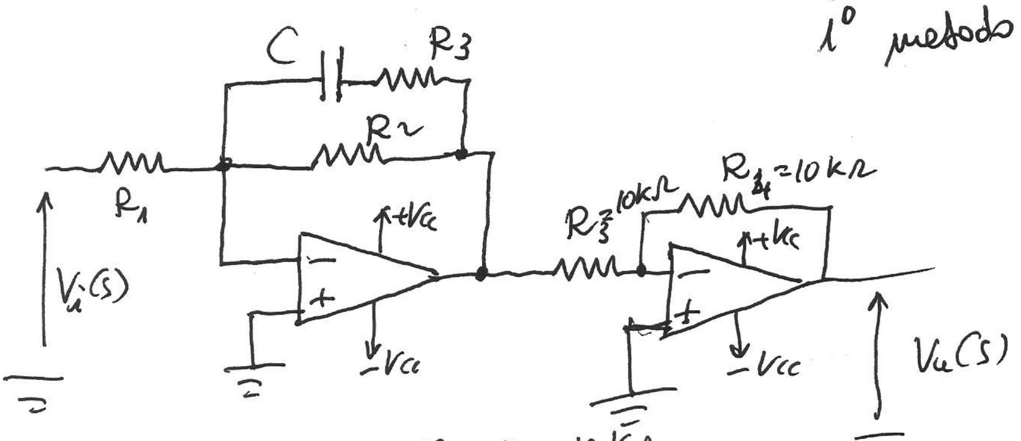


Carta semilogaritmica a 9 decadi
COMPENSAZIONE CON RETE RITARDATRICE
G(s) NON COMP. = 33,33 / ((1+s/6)(1+s/30)(1+s/200))
G_R(s) = (1+s/6) / (1+s/0,1)
RETE RITARDATRICE
G(s) COMP. = 33,33 / ((1+s/0,1)(1+s/30)(1+s/200))
K_st (dB) = 20 log10 33,33 = 30,45 dB
ωc ≈ 70 r/s
ωc' ≈ 3 rad/s
MG ≈ 36 dB
-20 dB/dec
-40 dB/dec
-60 dB/dec
dB
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
0,01
0,1
1
6
10
30
10²
200
10³
10⁴
10⁵
Pulsazione ω (rad/s)
ZERO
POLO
-45°/dec
-90°/dec
-135°/dec
MF ≈ 90°
90°
45°
0
-45°
-90°
-135°
-180°
-225°
-270°
-315°
-360°
MARGINE DI FASE MF ≈ 90° ; MARGINE DI GUADAGNO MG ≈ 36dB
21
Pulsazione ω (rad/s)

DIMENSIONAMENTO DELLA RETE RITARDATRICE ATTIVA.

1° metodo



$R_1 = R_2$; $R_4 = R_5 = 10\ k\Omega$

$$G_R(s) = \frac{R_3Cs+1}{(R_2+R_3)Cs+1} \quad ; \quad G_R(s) = \frac{1+\frac{s}{6}}{1+\frac{s}{0{,}1}}$$

$$\omega_z = \frac{1}{R_3C} \quad ; \quad \omega_p = \frac{1}{(R_2+R_3)C}$$

$$\frac{1}{R_3C} = 6\ rad/s$$

$$C = 1\ \mu F \quad ; \quad R_3 = \frac{1}{\omega_z C} = \frac{1}{6\cdot 10^{-6}} = 1{,}62\cdot 10^5\ \Omega \Rightarrow 180\ k\Omega$$

$$(R_2 + R_3)\ C = \frac{1}{\omega_p} = \frac{1}{0{,}1} = 10$$

$$R_2C + R_3C = 10$$

$$R_2 = \frac{10 - R_3C}{C} = \frac{10 - 1{,}62\cdot 10^5\cdot 10^{-6}}{10^{-6}} = 9{,}833\cdot 10^6\ \Omega \Rightarrow 10\ M\Omega$$

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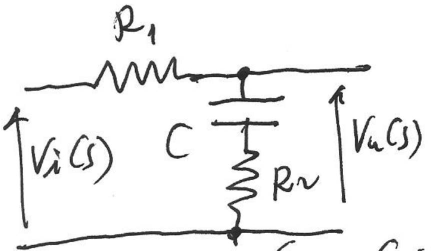
DIMENSIONAMENTO DELLA RETE RITARDATRICE PASSIVA

2° metodo.



$$V_o(s) = \frac{V_i(s)\left(R_2 + \frac{1}{sC}\right)}{R_1 + R_2 + \frac{1}{sC}} =$$

$$= \frac{V_i(s)\frac{(R_2 sC + 1)}{\cancel{sC}}}{\frac{(R_1+R_2)sC + 1}{\cancel{sC}}} = \frac{V_i(s)\,(R_2Cs + 1)}{1 + (R_1+R_2)Cs}$$

$$G_R(s) = \frac{V_u(s)}{V_i(s)} = \frac{R_2\cancel{C}\left(s + \frac{1}{R_2C}\right)}{(R_1+R_2)\cancel{C}\left(s + \frac{1}{(R_1+R_2)C}\right)}$$

$$\lim_{s\to 0} G_R(s) = 1$$

$$G_R(s) = \frac{1 + \frac{s}{6}}{1 + \frac{s}{0,1}}$$

$$\omega_z = \frac{1}{R_2C} = 6 \text{ rad/s} \quad ; \quad \omega_p = \frac{1}{(R_1+R_2)C} = 0,1 \text{ rad/s}$$

$$C = 1\,\mu F \; ; \quad R_2 = \frac{1}{\omega_z C} = \frac{1}{6 \cdot 10^{-6}} = 1,67 \cdot 10^5\,\Omega \Rightarrow 180\,k\Omega$$

$$(R_1+R_2)C = \frac{1}{0,1} = 10 \text{ rad/s}$$

$$R_1C = 10 - R_2C \; ; \quad R_1 = \frac{10 - R_2C}{C} =$$

$$= \frac{10 - 1,67\cdot 10^5 \cdot 10^{-6}}{10^{-6}} = 9,833 \cdot 10^6\,\Omega \simeq 10\,M\Omega$$

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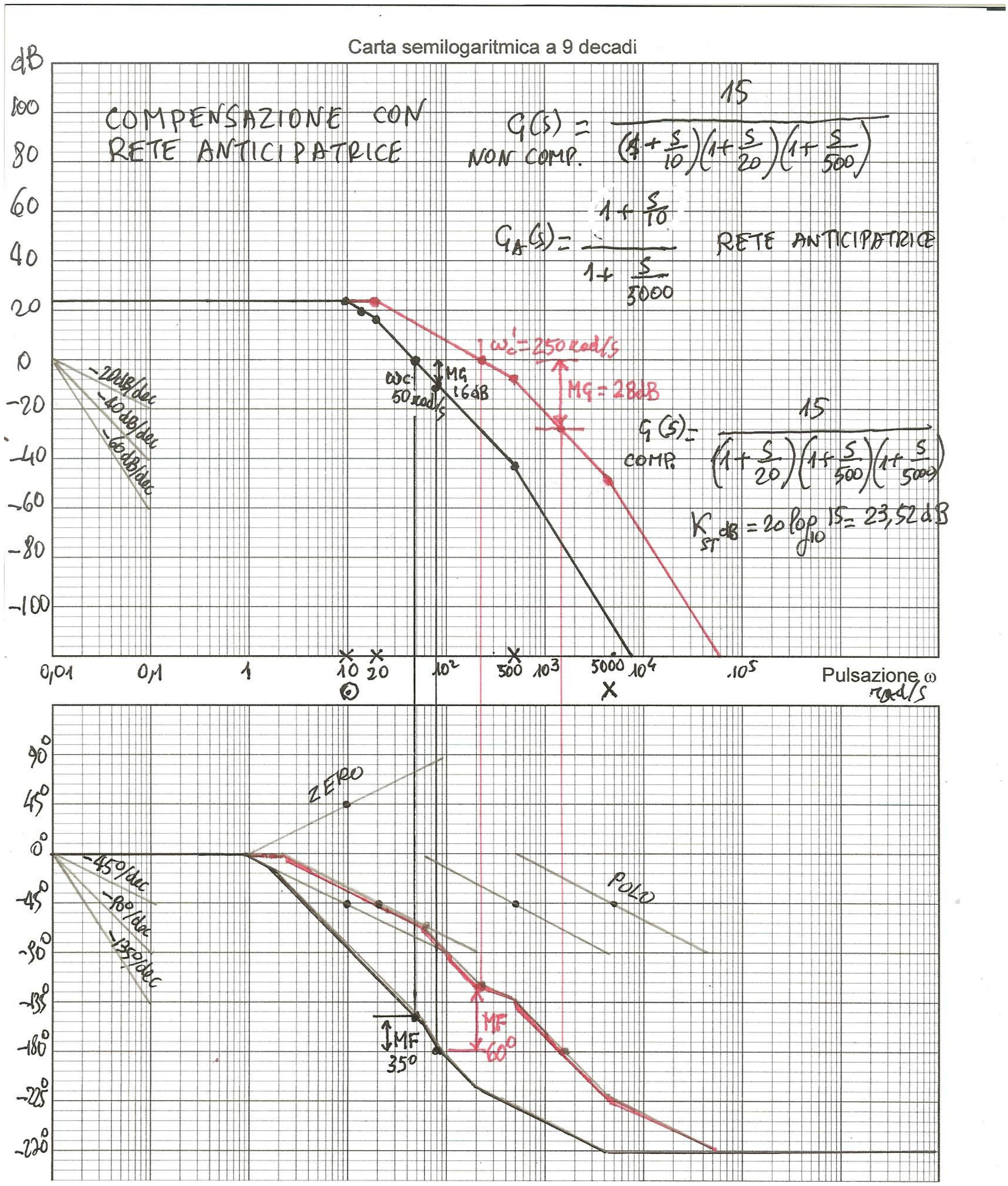

MARGINE DI FASE MF = 60°; MARGINE DI GUADAGNO MG = 28 dB ω (rad/s)

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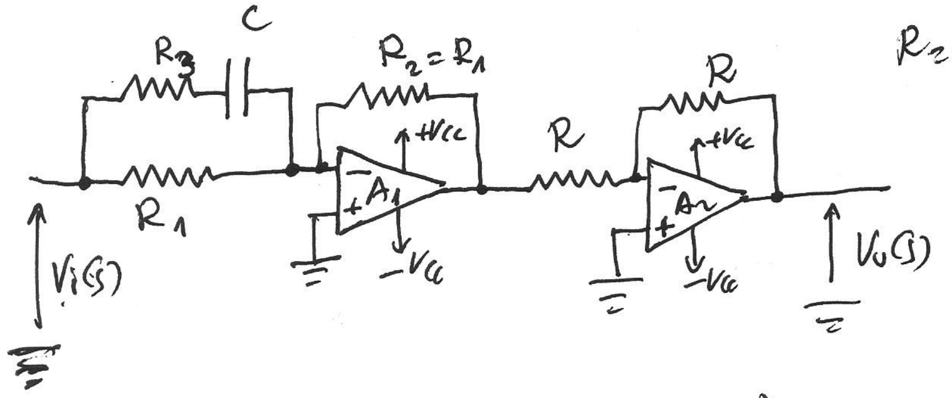
DIMENSIONAMENTO DELLA RETE ANTICIPATRICE ATTIVA

1° metodo

$R = 10\,k\Omega$



$$G_A(s) = \frac{V_o(s)}{V_i(s)} = \frac{1 + (R_1 + R_3)Cs}{1 + R_3 Cs}$$

$(R_1 + R_3)C = \tau_z$

$R_3 C = \tau_p$

$$G_A(s) = \frac{1 + \frac{s}{10}}{1 + \frac{s}{5000}}$$

$$\omega_z = \frac{1}{(R_1 + R_3)C} = \frac{1}{\tau_z}$$

$$\omega_p = \frac{1}{R_3 C} = \frac{1}{\tau_p}$$

$$\begin{cases} \dfrac{1}{(R_1 + R_3)C} = 10, \\ \dfrac{1}{R_3 C} = 5000 \end{cases}$$

$C = 100\,nF$; $\quad R_3 = \dfrac{1}{5000\,C} = \dfrac{1}{5 \cdot 10^3 \cdot 10^{-7}} = 2000\,\Omega = 2\,k\Omega$.

$(R_1 + R_3)C = 1/10 = 0{,}1$

$R_1 C = 0{,}1 - R_3 C$

$R_1 = R_2$

$$R_1 = \frac{0{,}1 - R_3 C}{C} = \frac{0{,}1 - 2 \cdot 10^3 \cdot 10^{-7}}{10^{-7}} \simeq \frac{0{,}1 - \cancel{2 \cdot 10^{-4}}}{10^{-7}} \simeq 10^6\,\Omega = 1\,M\Omega$$

Se $C = 1\,\mu F$ $\quad R_3 = \dfrac{1}{5 \cdot 10^3 \cdot 10^{-6}} = 200\,\Omega$

$$R_1 = \frac{0{,}1 - 2 \cdot 10^3 \cdot 10^{-6}}{10^{-6}} \simeq \frac{0{,}1 - \cancel{2 \cdot 10^{-3}}}{10^{-6}} \simeq 10^7\,\Omega = 10\,M\Omega$$

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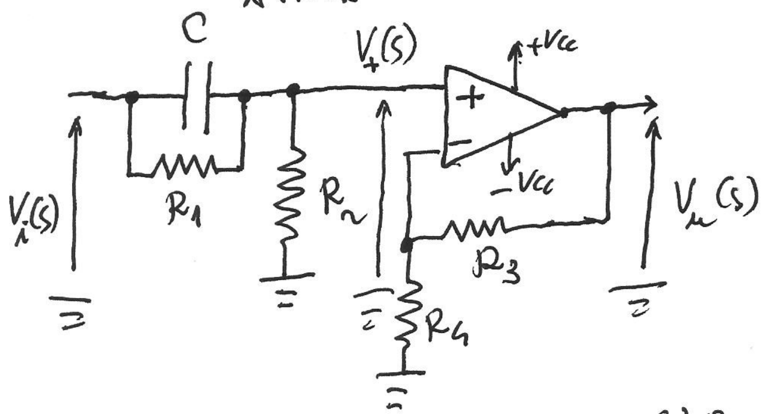
DIMENSIONAMENTO DELLA RETE ANTICIPATRICE ATTIVA

2° METODO



$$V_+(s) = \frac{V_i(s)\,R_2}{R_2 + R_1 /\!/ \frac{1}{sC}} = \frac{V_i(s)\,R_2}{R_2 + \frac{\frac{R_1}{sC}}{R_1 + \frac{1}{sC}}} = \frac{V_i(s)\,R_2}{R_2 + \frac{R_1}{R_1Cs+1}} =$$

$$= \frac{V_i(s)\,R_2\,(R_1Cs+1)}{R_1R_2Cs + R_2 + R_1} ; \qquad G_A(s) = \frac{V_u(s)}{V_i(s)} = \left(1 + \frac{R_3}{R_4}\right)\frac{V_+(s)}{V_i(s)}$$

$$\boxed{\lim_{s\to 0} \frac{V_+(s)}{V_i(s)} = \frac{1}{R_1C}\,\frac{R_1R_2C}{R_1+R_2} = \frac{R_2}{R_1+R_2}}$$

attenuazione.

$$\frac{V_+(s)}{V_i(s)} = \frac{\cancel{R_1R_2C}\left(s + \frac{1}{R_1C}\right)}{\cancel{R_1R_2C}\left(s + \frac{R_1+R_2}{R_1R_2C}\right)} = \frac{s + \frac{1}{R_1C}}{s + \frac{R_1+R_2}{R_1R_2C}}$$

$$G_A(s) = \left(1 + \frac{R_1}{R_2}\right)\frac{\left(s + \frac{1}{R_1C}\right)}{s + \frac{R_1+R_2}{R_1R_2C}}$$

$$\boxed{1 + \frac{R_3}{R_4} = \frac{R_1+R_2}{R_2} \qquad \frac{R_3}{R_4} = \frac{R_1+R_2}{R_2} - 1 = \frac{R_1+\cancel{R_2}-\cancel{R_2}}{R_2} =}$$

$$= \frac{R_1}{R_2}$$

$$G_A(s) = \frac{\frac{s}{10}+1}{1 + \frac{s}{5000}}$$

$$\omega_z = \frac{1}{R_1C} = 10\ \text{rad/s}$$

$$\omega_p = \frac{R_1 + R_2}{R_1R_2C} = 5000\ \text{rad/s}.$$

$$C = 100\ \text{nF} \qquad R_1 = \frac{1}{\omega_z C} = \frac{1}{10\cdot 10^{-7}} = 10^{6}\ \Omega = 1\ \text{M}\Omega$$

$$\left(\text{oppure } C = 1\ \mu\text{F} \qquad R_1 = \frac{1}{10\cdot 10^{-6}} = 100\ \text{k}\Omega\right)$$

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Con $C = 100\,nF$ e $R_1 = 1M\Omega$ si ha:

$$R_1 + R_2 = R_1 R_2 C \omega_p \quad ; \quad R_1 R_2 C\omega_p - R_2 = R_1 \; ;$$

$$R_2 (R_1 C\omega_p - 1) = R_1 ;$$

$$R_2 = \frac{R_1}{R_1 C\omega_p - 1} = \frac{10^6}{10^6 \cdot 10^{-7} \cdot 5 \cdot 10^3 - 1} = \frac{10^6}{499} = 2004\,\Omega$$

oppure, con $C = 1\,\mu F$ e $R_1 = 100\,K\Omega$

$$R_2 = \frac{R_1}{R_1 C\omega_p - 1} = \frac{10^5}{10^5 \cdot 10^{-6} \cdot 5 \cdot 10^3 - 1} = \frac{10^5}{499} = 200{,}4\,\Omega$$

$$\frac{R_3}{R_4} = \frac{R_1}{R_2} \implies$$ se $R_1 = 1M\Omega$ e $R_2 = 2004\,\Omega$
si ha: $R_3 = 1M\Omega$ e $R_4 = 2004\,\Omega$

oppure $\implies$ se $R_1 = 100k\Omega$ e $R_2 = 200{,}4\,\Omega$
si ha: $R_3 = 100k\Omega$ e $R_4 = 200{,}4\,\Omega$

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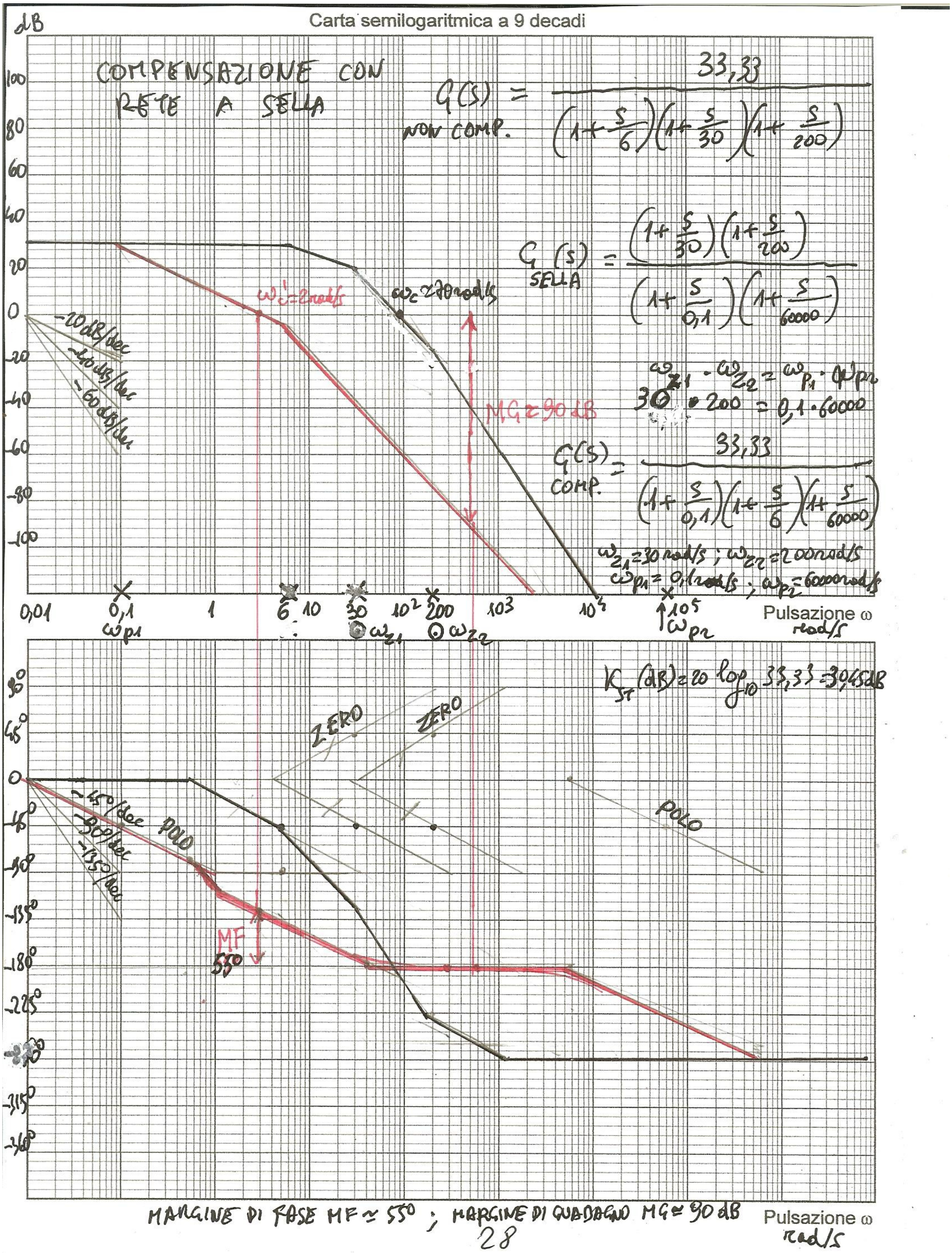
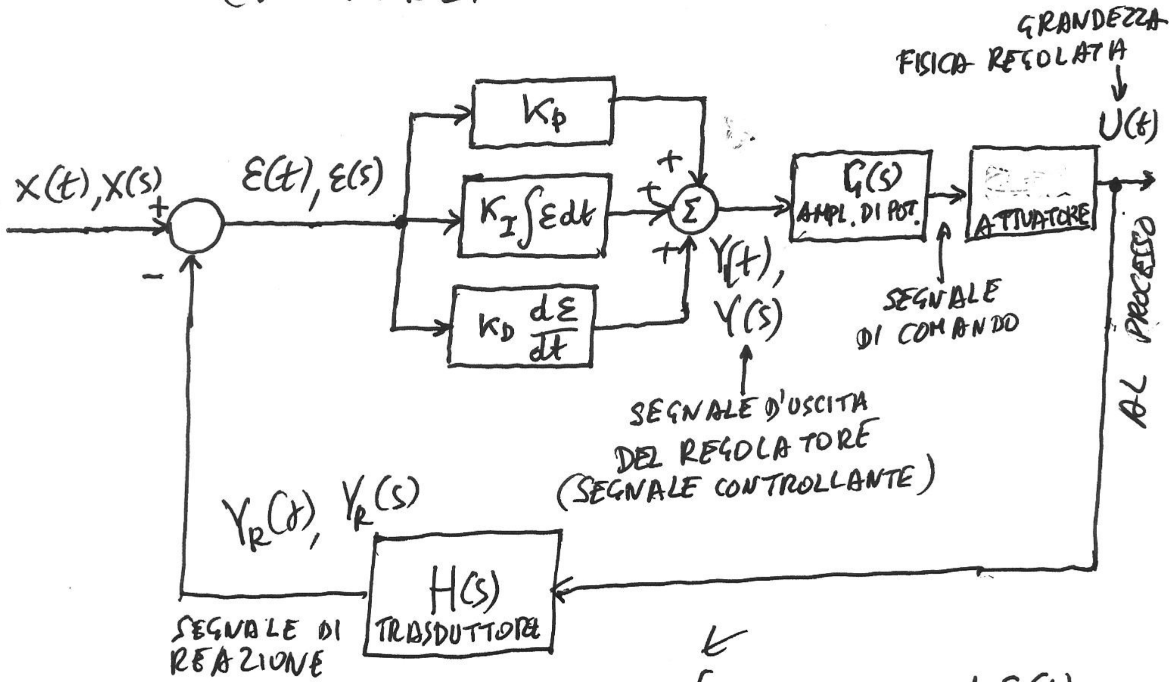

Carta semilogaritmica a 9 decadi
dB
COMPENSAZIONE CON RETE A SELLA
G(s) NON COMP. = 33,33 / ((1+s/6)(1+s/30)(1+s/200))
G(s) SELLA = ((1+s/30)(1+s/200)) / ((1+s/0,1)(1+s/60000))
ωz1 · ωz2 = ωp1 · ωp2
30 · 200 = 0,1 · 60000
G(s) COMP. = 33,33 / ((1+s/0,1)(1+s/6)(1+s/60000))
ωz1=30 rad/s ; ωz2=200 rad/s
ωp1=0,1 rad/s ; ωp2=60000 rad/s
ω'c=2 rad/s
ωc≈170 rad/s
MG≈90 dB
-20 dB/dec
-40 dB/dec
-60 dB/dec
0,01
0,1
ωp1
1
6
10
30
ωz1
10²
200
ωz2
10³
10⁴
10⁵
ωp2
Pulsazione ω rad/s
Kst (dB) = 20 log10 33,33 = 30,45 dB
ZERO
ZERO
POLO
POLO
-45°/dec
-90°/dec
-135°/dec
MF 55°
MARGINE DI FASE MF ≈ 55° ; MARGINE DI GUADAGNO MG ≈ 90 dB
28
Pulsazione ω rad/s

REGOLATORE PID
(PROPORZIONALE - INTEGRALE - DERIVATIVO



$$Y(t) = K_p\,\varepsilon(t) + K_I \int_0 \varepsilon(t)\,dt + K_D \frac{d\,\varepsilon(t)}{dt}$$

$$\varepsilon(t) = x(t) - Y_R(t)$$

set point

$$R(s) = \frac{Y(s)}{\varepsilon(s)} = K_p + \frac{K_I}{s} + K_D s =$$

FUNZIONE DI TRASFERIMENTO DEL REGOLATORE

$$= \frac{K_p s + K_I + K_D s^2}{s} = \frac{K_D\left(s^2 + \frac{K_p}{K_D}s + \frac{K_I}{K_D}\right)}{s} =$$

$$= \frac{K_D\left[s^2 + (z_1+z_2)s + z_1 z_2\right]}{s} = \quad ; \quad \frac{K_p}{K_D} = z_1 + z_2 \; ; \; \frac{K_I}{K_D} = z_1 z_2$$

$$= K_D \frac{(s+z_1)(s+z_2)}{s} = \frac{K_D z_1 z_2 \left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)}{s}$$

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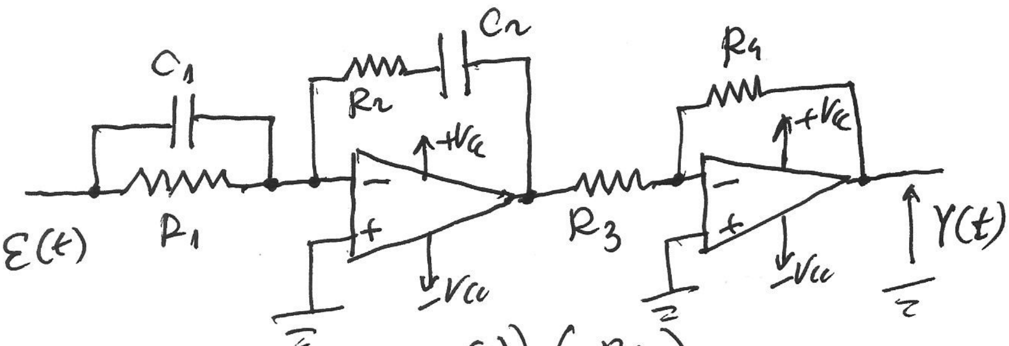
REGOLATORE PID



$$R(s) = \frac{Y(s)}{\varepsilon(s)} = \left(-\frac{Z_2(s)}{Z_1(s)}\right)\left(-\frac{R_4}{R_3}\right)$$

$$Z_2(s) = R_2 + \frac{1}{sC_2} = \frac{R_2C_2s+1}{sC_2}$$

$$Z_1(s) = R_1 // \frac{1}{sC_1} = \frac{R_1\frac{1}{sC_1}}{R_1+\frac{1}{sC_1}} = \frac{R_1}{1+R_1C_1s}$$

$$R(s) = \frac{R_4}{R_3}\cdot\frac{(R_2C_2s+1)}{sC_2}\cdot\frac{(1+R_1C_1s)}{R_1} =$$

$$= \frac{R_4}{R_1R_3C_2}\cdot\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)}{s}$$

$$z_1 = \frac{1}{R_1C_1} \quad ; \quad z_2 = \frac{1}{R_2C_2}$$

$$K_I = K_D z_1 z_2 = K_D \frac{1}{R_1C_1}\cdot\frac{1}{R_2C_2} \quad ; \quad K_P = K_D(z_1+z_2)$$

$$K_D z_1 z_2 = \frac{R_4}{R_1R_3C_2} \quad ; \quad K_D \frac{1}{\cancel{R_1}C_1}\cdot\frac{1}{R_2\cancel{C_2}} = \frac{R_4}{\cancel{R_1}R_3\cancel{C_2}} \quad ;$$

$$\boxed{K_I = \frac{R_4\cancel{C_1}\cancel{R_2}}{R_3}\cdot\frac{1}{R_1\cancel{C_1}}\cdot\frac{1}{\cancel{R_2}C_2} = \frac{R_4}{R_1R_3C_2}} \quad \boxed{K_D = \frac{R_4C_1R_2}{R_3}} \quad \boxed{K_P = \frac{R_4C_1R_2}{R_3}\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}\right)}$$

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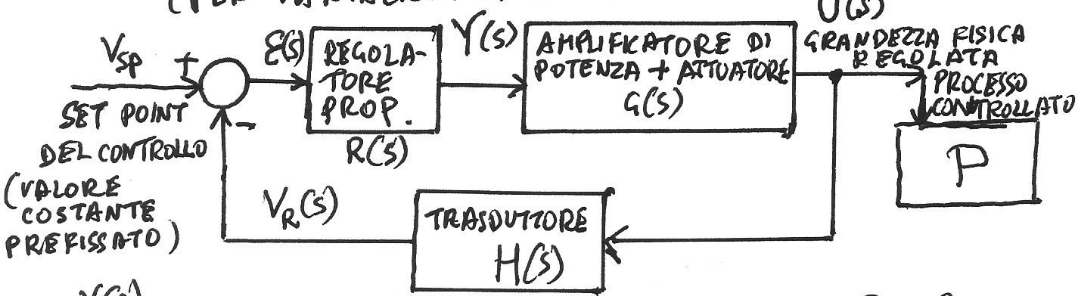
REGOLATORE PROPORZIONALE

(L'USCITA Y DIPENDE LINEARMENTE DALL'ERRORE ε)

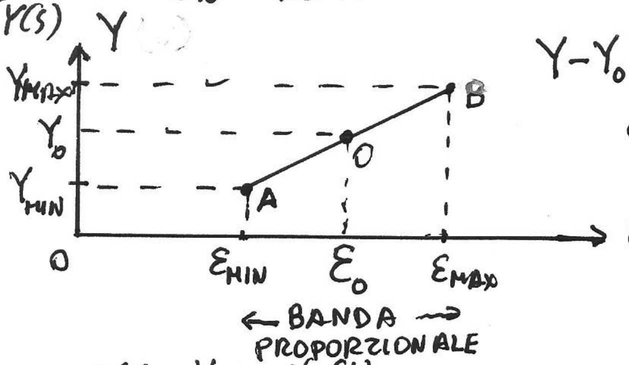
(PER VARIAZIONI DI CARICO DI PICCOLA AMPIEZZA)



$$R(s) = \frac{Y(s)}{\varepsilon(s)}$$

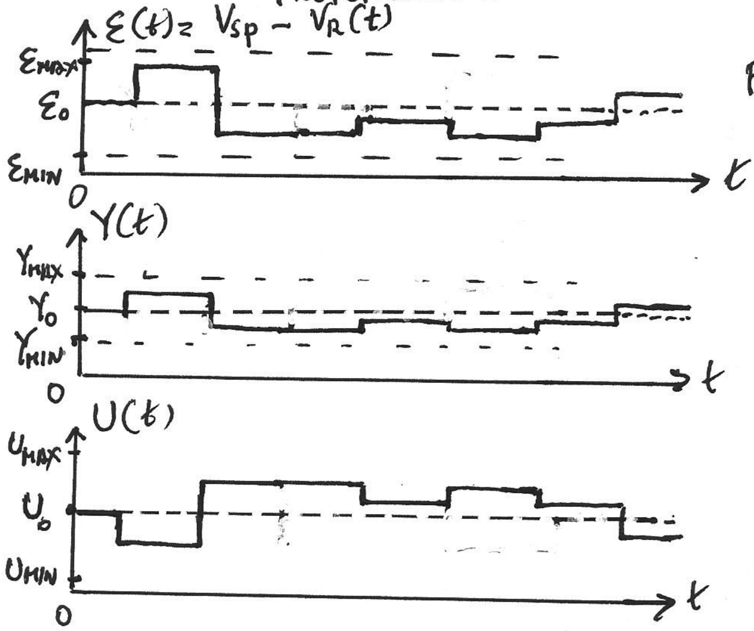
$$G(s) = \frac{U(s)}{Y(s)}$$

SCHEMA A BLOCCHI DI UN GENERICO SISTEMA DI CONTROLLO REALIZZATO CON UN REGOLATORE PROPORZIONALE



$$Y - Y_0 = P(\varepsilon - \varepsilon_0)$$

↑ COSTANTE DI PROPORZIONALITÀ DEL REGOLATORE



P = 0,6

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REGOLATORE PI (PROPORZIONALE-INTEGRALE)

1) $Y - Y_0 = P(\mathcal{E} - \mathcal{E}_0) + PI\int^t \mathcal{E}\,dt$

SI PONE $\quad I$ (COSTANTE DELL'INTEGRATORE $= \frac{1}{RC}$)
(FREQUENZA DI RIPETIZIONE)

$K_I = PI \qquad I = \frac{1}{T_I} \leftarrow$ TEMPO DI INTEGRAZIONE

SE IN UN DATO INTERVALLO DI TEMPO Δt LA VARIAZIONE $\Delta\mathcal{E} = \mathcal{E} - \mathcal{E}_0$ È COSTANTE, SI HA:

$$\Delta Y = P\Delta\mathcal{E} + PI\int^{\Delta t}\Delta\mathcal{E}\,dt = P\Delta\mathcal{E} + PI\Delta\mathcal{E}\Delta t;$$

L'AZIONE INTEGRALE PRODUCE UN EFFETTO UGUALE A QUELLO DELL'AZIONE PROPORZIONALE NELL'INTERVALLO Δt TALE CHE SIA $\cancel{P\Delta\mathcal{E}} = \cancel{P}I\cancel{\Delta\mathcal{E}}\Delta t$

$$1 = I\Delta t$$

$$\Delta t = \frac{1}{I} = T_I \text{ (IN MINUTI)}$$

L'AZIONE INTEGRALE PRODUCE UNA VARIAZIONE DI Y PARI A QUELLA PRODOTTA DALL'AZIONE PROPORZIONALE I VOLTE AL MINUTO

$\mathcal{L}$-trasformando l'equazione 1) si ha:

$$Y(s) - \frac{Y_0}{s} = P\mathcal{E}(s) - \frac{P\mathcal{E}_0}{s} + \frac{PI\mathcal{E}(s)}{s}$$

ESSENDO $\quad Y - Y_0 = P(\mathcal{E} - \mathcal{E}_0), \quad Y_0 = P\mathcal{E}_0,$

SI HA: $\quad Y(s) = \underbrace{P\left(1 + \frac{I}{s}\right)}\mathcal{E}(s)$

$R(s) = \frac{Y(s)}{\mathcal{E}(s)} = P\left(1 + \frac{I}{s}\right) = \boxed{R(s)}$ FUNZIONE DI TRASFERIMENTO DEL REGOLATORE PI

$$= \frac{P(s + I)}{s} = \frac{K_I}{\cancel{I}s}\cancel{I}\left(\frac{s}{I} + 1\right) = \frac{K_I}{s}\left(1 + \frac{s}{I}\right)$$ SISTEMA DI TIPO 1

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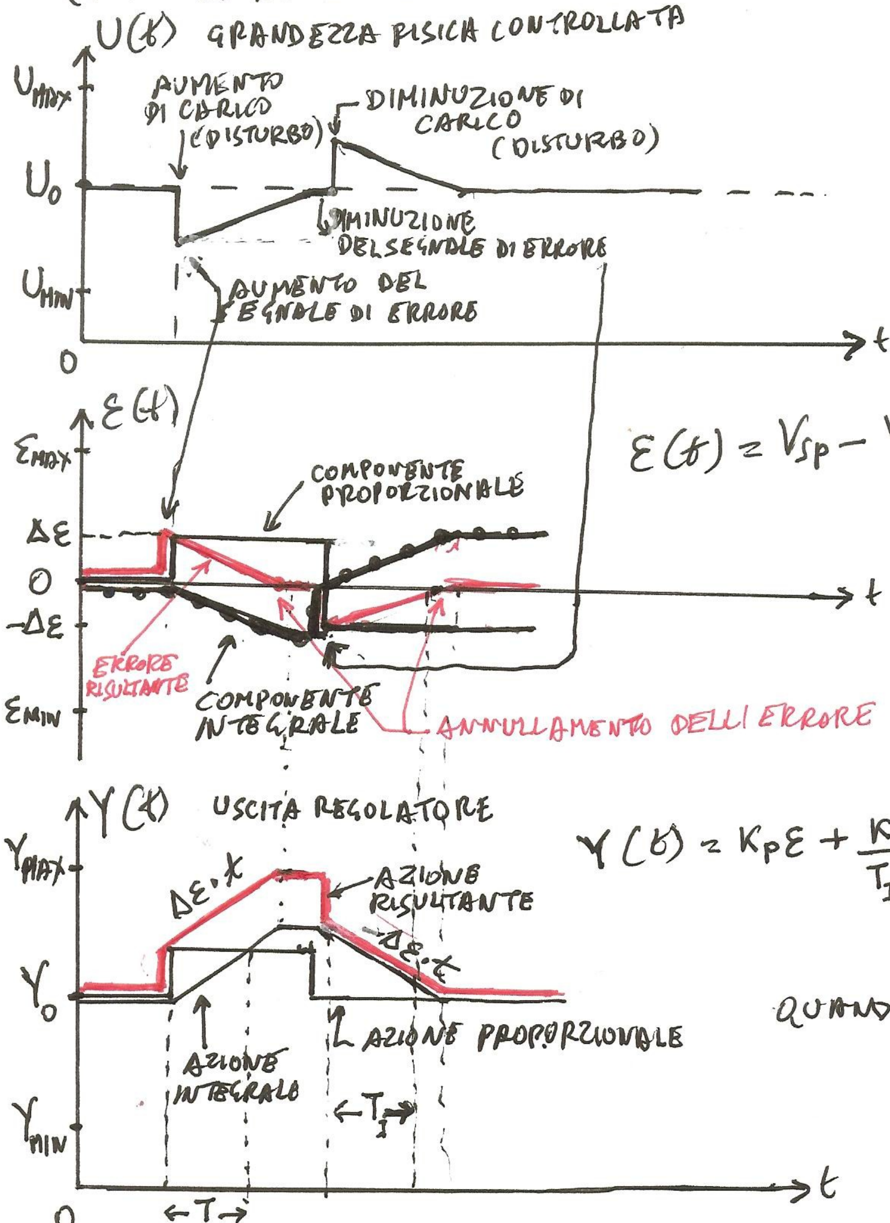
AZZERAMENTO DELL'ERRORE DI POSIZIONE PER EFFETTO DELL'AZIONE INTEGRALE AGGIUNTA ALL'AZIONE PROPORZIONALE
(PER VARIAZIONI DI CARICO DI GRANDE AMPIEZZA)



$$\varepsilon(t) = V_{sp} - V_R(t)$$

$$Y(t) = K_P \varepsilon + \frac{K_P}{T_I} \int_0^t \underset{\Delta\varepsilon}{\varepsilon}\, dt + \underset{Y_0}{\text{cost.}}$$

QUANDO $\varepsilon \to 0$

$Y \to Y_0$

$$T_I = \frac{1}{I}$$

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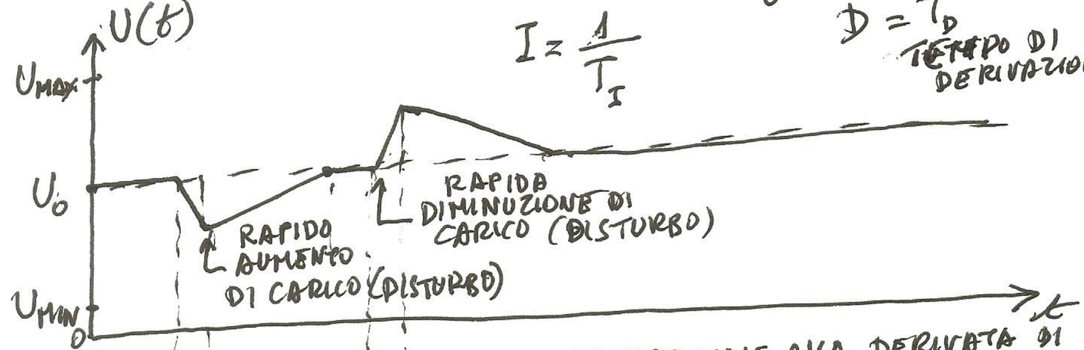
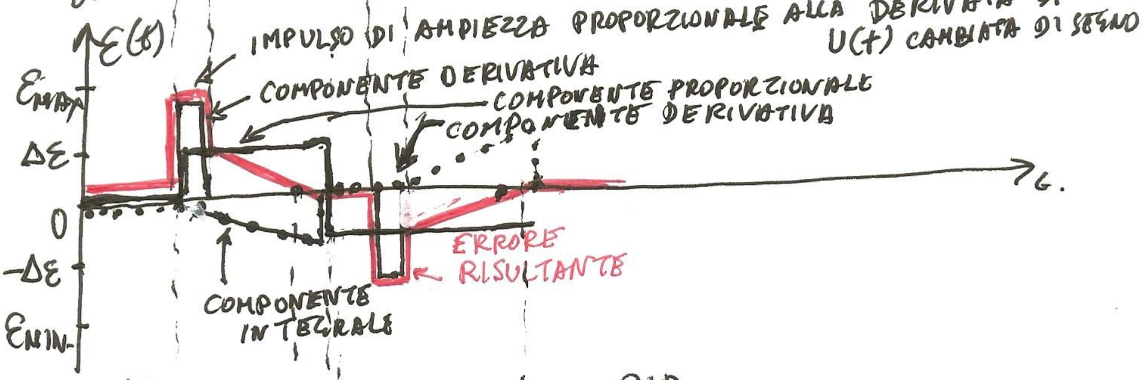
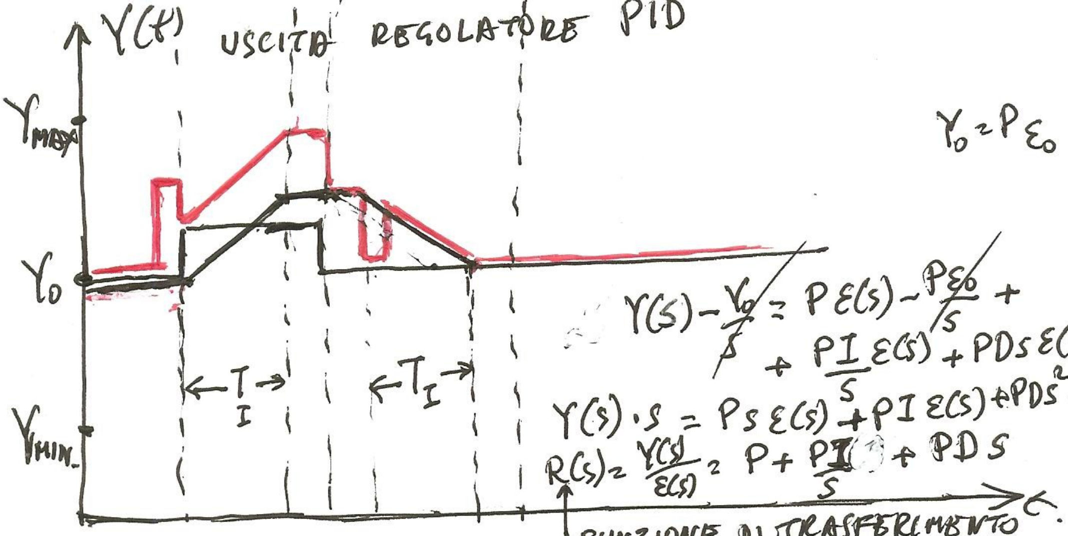
REGOLATORE PID
(PROPORZIONALE – INTEGRALE – DERIVATIVO)
PER RAPIDE VARIAZIONI DI CARICO DI GRANDE AMPIEZZA

$$Y(t) - Y_0 = P\left(\varepsilon(t) - \varepsilon_0\right) + PI\int_0^t \varepsilon(t)\,dt + PD\,\frac{d\varepsilon(t)}{dt}$$

$I = \dfrac{1}{T_I}$ $\qquad D = T_D$ TEMPO DI DERIVAZIONE







$Y_0 = P\varepsilon_0$

$$Y(s) - \cancel{\frac{Y_0}{s}} = P\varepsilon(s) - \cancel{\frac{P\varepsilon_0}{s}} + \frac{PI}{s}\varepsilon(s) + PDs\,\varepsilon(s)$$

$$Y(s)\cdot s = Ps\,\varepsilon(s) + PI\,\varepsilon(s) + PDs^2\varepsilon(s)$$

$$R(s) = \frac{Y(s)}{\varepsilon(s)} = P + \frac{PI}{s} + PDs$$

FUNZIONE DI TRASFERIMENTO DEL REGOLATORE PID

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